Granular Search in the Product Market

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Abstract

This paper studies the implications of the emergence of dominant firms for aggregate market power and the growth of small fringe firms. The greater *visibility* of large sellers endows them with a matching advantage in the product market which they use to charge high markups. Long-run competitive forces discipline the behaviour of a granular seller and in a natural benchmark model firm dynamics of fringe firms are neutral under the presence of a large seller. I then present an extension in which the presence of the granular seller raises aggregate markups, lowers the growth of the most productive firms and slows down the reallocation of customers from unproductive to productive sellers. This sheds light on a new mechanism of how dominant firms could shape the aggregate behaviour of the macroeconomy.

Keywords: Market Power, Granular Firms, Product Market Search, Business Dynamism

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1 Introduction

The increase in product-market concentration and the rise of superstar firms have emerged as a robust empirical finding in the U.S. over at least the past 40 years (Autor et al. 2020, Kwon et al. 2022).[1] While the welfare implications of rising concentration are theoretically ambiguous (Syverson 2019*a*,*b*), greater product market concentration has been linked to several macroeconomic trends.[2] Two such coinciding trends have been the slowdown of business dynamism and the rise of markups. For example, Decker et al. (2016) highlight the marked decline in high-growth young firms since 2000, a trend which is particularly concerning as high-growth young firms are found to be an important contributor to sustained job creation. On the rise of market power, De Loecker et al. (2020) document an increase in aggregate markups from 1.2 to 1.6 over the past 40 years. A natural question is how these trends are related and if the emergence of superstar firms could have upended the structure of the macroeconomy.

In this paper, I explore the role of the emergence of dominant granular firms as a potential driver for the rise in market power and the changing nature of firm dynamics of smaller fringe sellers.[3] The hypothesis of the paper states that the presence of granular firms raises aggregate markups and may cause a decline in high-growth fringe sellers, though long-run forces discipline the magnitude of these effects in steady state. The key mechanism is as follows. Product markets are subject to search frictions. Due to their granularity large sellers are hard to avoid and match with customers more easily. In contrast, as smaller fringe sellers are infinitesimal they are harder to find. This endows granular sellers with additional market power, which they use to raise markups.

To build intuition, I first introduce a simple static model of granular search in the product market. I establish a direct mapping between the granularity of the large seller and its customer base and formally characterise the additional market power derived from such granularity. I extend the framework to a tractable dynamic general equilibrium model in order to speak to the inherently intertemporal nature of the decline in high-growth young firms. An important insight of the baseline dynamic model is that granularity is not an exogenous object but determined in equilibrium and disciplined by long-run competitive forces. Only if the large seller is sufficiently productive will they maintain their granularity in the steady state.

As a further implication of the disciplining forces of the long-run, I derive an approximate neutrality result in which the steady state pricing behaviour of fringe sellers is identical in a model with versus a model without a granular seller, even though fringe sellers are scaled down in size. This result is a direct

[1]There are some questions, however, on how to measure concentration or how to interpret concentration measures. Rossi-Hansberg et al. (2021) argue that the relevant concentration measure defines markets locally, where concentration has even fallen, and not nationally. Gutiérrez & Philippon (2020) argue that sales of dominant shares should be measured against global and not national sales.

[2]Covarrubias et al. (2020) find evidence of a turn from efficient to inefficient concentration from 2000 onwards. Autor et al. (2020) link rising concentration of sales in most US sectors to a decline in the labour share. Barkai (2020) additionally document a rise in the profitability of firms. Gutiérrez & Philippon (2017) relate rising concentration to a fall in investment.

[3]A granular seller is defined as a firm with positive mass as opposed to infinitesimal fringe sellers.

implication of the steady state condition of the customer base of the granular seller.

In the final section, I present a modification to the benchmark model that establishes a direct link between the emergence of a large, granular seller and the decline in high-growth young firms. While the benchmark specification assumes that the matching advantage endowed by granularity perfectly scales with the customer base, I then adopt an alternative formulation similar to Menzio & Trachter (2015) in which the matching advantage is constant. The extension yields novel results. In the presence of a granular firm, fringe sellers are harder to find, find it more difficult to attract new customers, and thus see their customer investment motives reduced. As a result, productive firms find it more costly to expand causing their growth rates to fall. This introduces the novel insight that the granularity of firms interacts with the customer flows between firms.

The primary difference to the benchmark model lies in the fact that the *visibility* of the large seller no longer scales in the customer base. Whereas in the benchmark model the granular seller can only raise markups at the expense of a lower matching advantage in the future, no such trade-off is present in the alternative model. In reality, the matching advantage of a granular seller is most likely determined by both, the customer base and a collection of other attributes such as advertising or appeal.

The model in this paper preserves many empirically plausible features of variable markup models such as Atkeson & Burstein (2008). For example, markups are in general increasing in the market share and the productivity of the large seller. However, this paper differs qualitatively from existent models of variable markups and firm dynamics in important ways. First, search frictions in product markets place dynamics at the centre. An important driver of firm growth in the model is the ease with which fringe sellers are able to attract customers in the future. Granular sellers may interfere with this channel as they exert a gravitational pull and divert customers from smaller fringe sellers. I therefore identify a new mechanism of how large firms could shape the aggregate behaviour of the macroeconomy based on distorting the reallocation of customer towards high productivity firms. Whether these forces are relevant in the steady state depends on the specific matching process. Second, in standard models the introduction of a high productivity firm unambiguously increases competition and lowers prices. In contrast, the model in this paper highlights that the interaction between search frictions and a high markup granular seller can induce *price-increasing* competition in which the high prices of the granular seller lifts the overall price level in the economy.

Related Literature. This paper is closest to Bornstein (2021) and Paciello et al. (2019). The first paper studies the role of an ageing economy as a driver for the observed joint phenomena of a decline in entry and rise in profitability in a related model of product market search. The second paper explores the sensitivity of shopping behaviour to pricing decisions of firms and uses a similar matching process between customers and sellers as in this paper. The model developed in this paper is an extension of both papers to feature a strategic, granular firm.

A wider literature emphasises the role of intertemporal investment motives of firms and the relative importance of demand factors as a bottleneck for firm growth (Gourio & Rudanko 2014, Foster et al. 2016, Sedláček & Sterk 2017, Roldan-Blanco & Gilbukh 2021, Argente et al. 2021, Nakamura & Steinsson 2011). To my best knowledge this paper is the first to study the implications of granularity in models of product market search for both the aggregate economy and the growth of small fringe firms.

This paper also speaks to the granular firm literature sparked in macroeconomics by Gabaix (2011). As a notable example, Carvalho & Grassi (2019) explore firm dynamics with large firms. Whereas the literature primarily focuses on the role of granularity as a driver for aggregate fluctuations this paper studies the implications of the presence of a large seller for the dynamics of smaller firms, even if the state variables of the granular firm are constant over time.

The role of granularity in labour market search is shown in Jarosch et al. (2021) to be of empirical importance for the wage setting behaviour of firms. Different to that paper and to reflect the context of product markets, I do not model price setting as the outcome of a bargaining process but as standard price posting by firms à la Bertrand. Further, the effects of granularity follow naturally from the assumed matching process and do not rely on firms using credible threats. This paper is also related to Berger et al. (2022), who stress the interaction of market power with labour market flows. This paper uses a similar idea for product market flows.

Outline. The paper is structured as follows. Section 2 presents a simple static model to build intuition. Section 3 extends the framework to a dynamic setting and characterises the dynamic behaviour of markups in a stationary equilibrium. Section 4 presents the neutrality result and a model extension that breaks it. Section 5 concludes. Model derivations, omitted proofs, and additional results are contained in the Appendix.

2 Static Model

This section introduces a simple static model to build intuition. On the consumption side, the economy features a unit measure of households $h \in [0, 1]$. On the production side, there is a single large, granular seller indexed by l and a continuum of identical fringe firms indexed by $j \in [0, J]$. Firms compete on prices subject to search frictions in the product market. The interaction between search frictions and granularity endows the large seller with additional market power.

2.1 Preferences

Each household $h \in [0,1]$ has nominal income I that they spend to purchase a single good from a single seller. As the model is static and preferences are assumed to satisfy the monotonicity property each household spends all of their income on one good. For example, if household h buys good k at price p_k they consume

$$c_k^h = \frac{I}{p_k}.$$

As in Bornstein (2021) preferences of household h for consuming good k are given by

$$U_k^h = u\left(\exp\left(\frac{1}{\sigma}\epsilon_k^h\right) c_k^h\right),$$

where $u(\cdot)$ denotes a strictly increasing function and ϵ_k^h a household-good specific taste shock and is drawn iid across h and k. The taste heterogeneity ensures that demand schedules aggregated over households are well-behaved. Motivated by the discrete choice literature in IO, I assume that the taste shock follows the Gumbel distribution.[4] This specification implies that the indirect utility of household h that chooses amongst a subset of goods $\mathcal{C}_h \subseteq [0, J] \cup \{l\}$ is given by

$$v^h = \max_{k \in \mathcal{C}_h} -\sigma \log(p_k) + \epsilon_k^h. \tag{2.1}$$

The model deviates from Bornstein (2021) in the matching process in the product market and the determination of the choice set $\mathcal{C}_h$.

2.2 Matching Technology in Product Market

Each firm competes for customers on prices and is subject to search frictions in the product market. Search frictions place restrictions on the choice set $\mathcal{C}_h$ from which each household h chooses their utility maximising product. At any given time households can only sample a finite number of alternative sellers.

Search proceeds as follows. First, at the beginning of the period each household h is initially matched to a seller indexed by $i_h \in \{l\} \cup [0, J]$ – either to the granular seller l or to one of the fringe sellers $j \in [0, J]$. An exogenous fraction $1 - \theta$ does not search and purchase the good of the firm they are initially matched with as in Bornstein (2021). Formally, for all households $\tilde{h}$ that do not search their choice set is the singleton $\mathcal{C}_{\tilde{h}} = \{i_{\tilde{h}}\}$. A fraction θ search and randomly meet *one other* seller.[5] The matching probability with a seller is equal to the initial customer base of that seller. For all households $\bar{h}$ that do search their choice set becomes $\mathcal{C}_{\bar{h}} = \{i_{\bar{h}}\} \cup \{i_r\}$ where the index i_r is randomly drawn from the collection of indices of all

[4] A random variable that follows a Gumbel distribution has cdf $F(x) = e^{-e^{-x}}$.

[5] The exogenous search probability ensures that the Diamond paradox (Diamond 1971) does not apply here. We could interpret θ as the fraction of households with zero search cost and $1 - \theta$ as the fraction with infinite search cost.

firms in the economy according to a discrete-continuous distribution over indices. Formally, without loss of generality, let the index for the large seller be $l > J$ and define the cumulative distribution over indices as

$$F(i_r) = \begin{cases} 0 & \text{if } i_r < 0, \\ \int_0^{i_r} m(j)dj & \text{if } 0 \leqslant i_r \leqslant J, \\ 1 - m_l & \text{if } J < i_r < l, \\ 1 & \text{if } i_r \geqslant l, \end{cases} \tag{2.2}$$

where m_l denotes the initial customer base of the large seller and $m(j)$ the initial customer base of fringe seller j. The cumulative density function in (2.2) characterises the matching process of a searching customer with a new seller index by i_r. The formulation implies that the meeting probability and meeting rate are equal to the initial customer base for the granular and fringe sellers, respectively. That is, a searching customer matches with a fringe seller j at a rate $m(j)$ and with the granular seller with a probability m_l.

Note the symmetry in the matching process between a fringe seller and a granular seller. This ensures that irrespective of their granularity sellers find it easier to attract customers the larger they are. The only difference to a fringe seller stems from the fact that customers match with a granular seller with positive probability. Once search has concluded, end of period customer bases are determined and sales take place.

The interpretation of the above matching process is information diffusion by word of mouth as in Rob & Fishman (2005) and Paciello et al. (2019). Customers sample purchase recommendations from each other and switch from their initial seller if the alternative is sufficiently attractive, both in terms of the idiosyncratic taste shock and the price. The customer base of a seller may also be a reasonable approximation for other measures of prominence relevant for product market search, which is captured above in a reduced form way. There is also a close parallel between the matching process in the present paper and the technology diffusion literature (Lucas & Moll 2014).

2.2.1 Granular Seller

The matching process for the granular seller is illustrated in Figure 1. The granular seller starts with m_l initially attached customers. A measure $m_l(1-\theta)$ stay attached and directly purchase from the large seller. The remaining $m_l\theta$ detach and search. Denote by $\mathcal{O}_l$ the fraction of those customer that find the new match more attractive and switch to a fringe sellers. Then, the granular seller retains $m_l\theta(1-\mathcal{O}_l)$ of initially attached customers that search. At the same time, $(1-m_l)\theta$ customers initially matched to fringe sellers detach and search. From each fringe seller j the granular seller attracts a fraction $\mathcal{I}_l(j)$ of the $m(j)\theta$

searching customers. Hence, the granular seller's end of period customer base is given by

$$\mathcal{M}_l = \underbrace{m_l(1-\theta)}_{\text{attached}} + \underbrace{m_l\theta(1-\mathcal{O}_l)}_{\text{retained}} + \underbrace{\int_0^J m(j)\theta\mathcal{I}_l(j)dj}_{\text{attracted}}.$$

Lemma 1. *Given the assumed structure of the matching process in the product market, consumer preferences and the Gumbel distribution assumption on the taste shocks, we get*

$$\mathcal{O}_l = \int_0^J m(j)\frac{(p_l/p_f(j))^\sigma}{1+(p_l/p_f(j))^\sigma}dj, \tag{2.3}$$

$$\mathcal{I}_l(j) = m_l\frac{1}{1+(p_l/p_f(j))^\sigma}, \tag{2.4}$$

where the granular seller charges price p_l, fringe sellers $p_f(j)$ for all $j \in [0, J]$ and m_l and $m(j)$ denote the initial customer base of the large seller and fringe firm j, respectively, with $\int_0^J m(j)dj = 1 - m_l$. In a symmetric equilibrium in which fringe sellers charge the same price, i.e. $p_f(j) = p_f$ for all $j \in [0, J]$

$$\mathcal{M}_l(p_l, p_f) = m_l - \underbrace{m_l\theta(1-m_l)\frac{(p_l/p_f)^\sigma}{1+(p_l/p_f)^\sigma}}_{\textit{Outflows}} + \underbrace{(1-m_l)\theta m_l\frac{1}{1+(p_l/p_f)^\sigma}}_{\textit{Inflows}}. \tag{2.5}$$

Proof. See Appendix B.1. □

Lemma 1 highlights the role of the granularity assumption. First, with respect to outflows a searching customer initially matched to the granular seller *re*-matches with the granular seller with probability m_l and purchases directly from the granular seller. It is in this sense that a fringe seller is harder to find. Hence, the large seller effectively only competes to retain a fraction $\theta(1-m_l)$ instead of θ of their initially attached customers m_l. In other words, granularity lowers the effective Calvo probability with which customers detach from a large seller to $\hat{\theta} = (1-m_l)\theta < \theta$.

Further, the pool of potential customers who the large seller can attract, given by $(1-m_l)\theta$, shrinks as the initial customer base increases. As we will see below, both factors reduce the incentive of granular sellers to compete on price and increase their incentives to raise markups.

2.2.2 Fringe Sellers

The matching process for a fringe seller j is illustrated in Figure 2. A fringe seller $j \in [0, J]$ starts with $m(j)$ customers initially attached to it. A fraction $m(j)(1-\theta)$ stays attached and directly purchase from the fringe seller j while the fringe seller j manages to retain the fraction $1-\mathcal{O}_f(j)$ of searching customers $m(j)\theta$. Inflows into the fringe seller j's customer base now come from two sources. From the $(1-m_l)\theta$ searching customers initially attached to another fringe seller and from the $m_l\theta$ searching customers initially attached

Granular Seller:

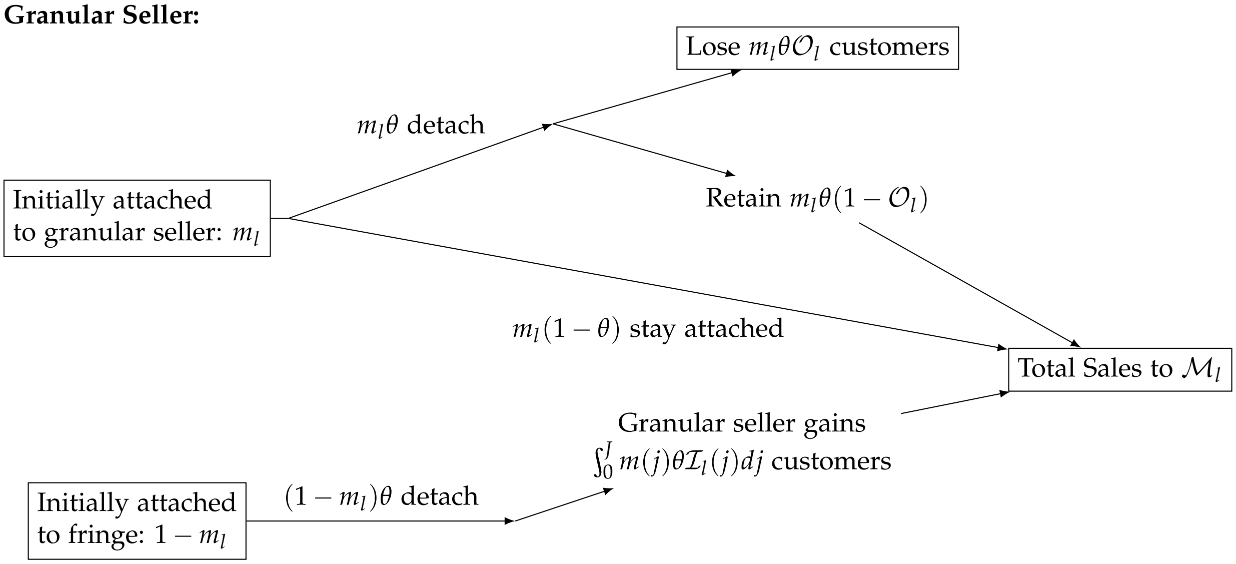

Figure 1: Matching Process for the Granular Seller.

Fringe Sellers:

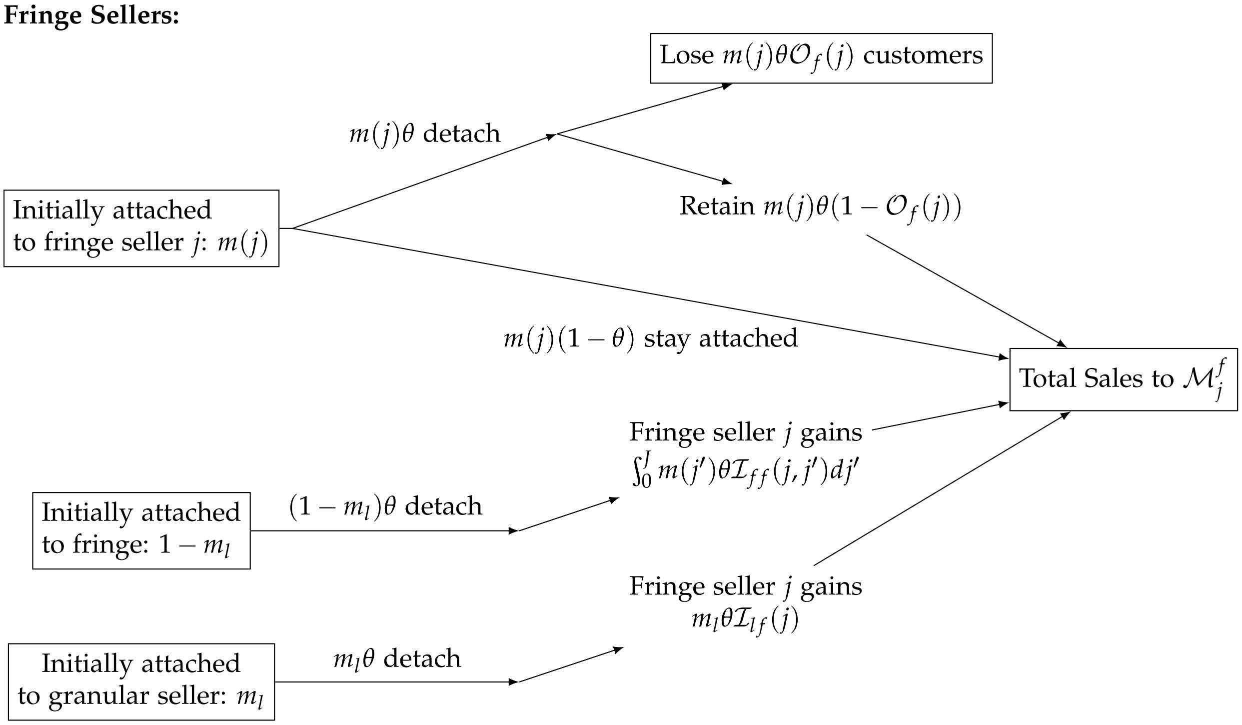

Figure 2: Matching Process for a Fringe Seller j.

to a granular seller. From each fringe seller j' the fringe seller j attracts a fraction $\mathcal{I}_{ff}(j, j')$ of the $m(j')\theta$ searching customers of fringe seller j'. Of the searching customers $m_l\theta$ of the large seller the fringe seller j attracts a fraction $\mathcal{I}_{lf}(j)$. Hence, the end of period customer base of fringe seller j is given by

$$\mathcal{M}^f_j = \underbrace{m(j)(1-\theta)}_{\text{attached}} + \underbrace{m(j)\theta(1-\mathcal{O}_f(j))}_{\text{retained}} + \underbrace{\int_0^J m(j')\theta\mathcal{I}_{ff}(j,j')dj' + m_l\theta\mathcal{I}_{lf}(j)}_{\text{attracted}}. \tag{2.6}$$

Lemma 2. *Given the assumed structure of the matching process in the product market, consumer preferences and the Gumbel distribution assumption on taste shocks, we get*

$$\mathcal{O}_f(j) = m_l \frac{(p_f(j)/p_l)^\sigma}{1+(p_f(j)/p_l)^\sigma} + \int_0^J m(j') \frac{(p_f(j)/p_f(j'))^\sigma}{1+(p_f(j)/p_f(j'))^\sigma} dj', \tag{2.7}$$

$$\mathcal{I}_{ff}(j,j') = m(j)\frac{1}{1+(p_f(j)/p_f(j'))^\sigma}, \tag{2.8}$$

$$\mathcal{I}_{lf}(j) = m(j)\frac{1}{1+(p_f(j)/p_l)^\sigma}, \tag{2.9}$$

where fringe seller j charges price $p_f(j)$, the granular seller p_l, and other fringe sellers $p_f(j')$ for all $j' \in [0, J]$. In a symmetric equilibrium in which all other fringe sellers charge the same price p_f the end of period customer base of fringe seller j is

$$\mathcal{M}^f_j(p_f(j), p_f, p_l) = m(j)\Delta^f_j(p_f(j), p_f, p_l) \tag{2.10}$$

where

$$\Delta^f_j(p_f(j), p_f, p_l) = 1 - \underbrace{\theta m_l \frac{(p_f(j)/p_l)^\sigma}{1+(p_f(j)/p_l)^\sigma}}_{\text{Outflows } j\mapsto l} + \underbrace{m_l\theta\frac{1}{1+(p_f(j)/p_l)^\sigma}}_{\text{Inflows } l\mapsto j}$$
$$- \underbrace{\theta(1-m_l)\frac{(p_f(j)/p_f)^\sigma}{1+(p_f(j)/p_f)^\sigma}}_{\text{Outflows } j\mapsto j',\ j'\in[0,J]} + \underbrace{(1-m_l)\frac{1}{1+(p_f(j)/p_f)^\sigma}}_{\text{Inflows } j'\mapsto j,\ j'\in[0,J]} \tag{2.11}$$

Proof. See Appendix B.2. ☐

Lemma 2 underscores the importance of the granularity assumption. With respect to outflows, a searching customer initially matched to fringe seller j *re*-matches with j with probability zero as they are infinitesimal. With respect to inflows, the pool of potential customers who the fringe seller can attract is θ as opposed to $(1 - m_l)\theta$ as is the case for the granular seller. This comes from the fact that fringe sellers can attract customers of both, other fringe sellers and the granular seller. In summary, relative to the granular seller a fringe seller finds it harder to retain customers while at the same time it considers a greater pool of potential customers. Both forces increase incentives to lower prices relative to the granular seller.

2.3 The Firm Problem

Each firm in the economy sells a single good, uses a constant returns to scale technology in labour and takes the same input cost for labour w as given. The granular seller has productivity A_l. All fringe sellers have the same productivity A_f, which I allow to be different from A_l. Firms compete on prices à la Bertrand.[6]

Granular Seller. The total demand faced by the large seller is given by the product of their customer base and their sales per customer. As households spend their entire income on one good the total demand faced by the large seller is given by $d_l(p_l, \hat{p}_f) = \mathcal{M}_l(p_l, \hat{p}_f)\frac{I}{p_l}$ for a given conjecture of the (symmetric) equilibrium price of fringe sellers $\hat{p}_f$. Hence, the large seller solves

$$\begin{aligned} V_l(m_l) = & \max_{p,y} y\left(p - \frac{w}{A_l}\right) \\ & \text{s.t.} \quad y \leqslant \mathcal{M}_l(p, \hat{p}_f)\frac{I}{p} \\ & \mathcal{M}_l(p, \hat{p}_f) = m_l\left(1 + (1-m_l)\theta\frac{1-(p_l/\hat{p}_f)^\sigma}{1+(p_l/\hat{p}_f)^\sigma}\right). \end{aligned}$$

where the objective is to maximise static profits, the first constraint captures the demand constraint and the second defines the law of motion for the end of period customer base.

Proposition 1. *Given a conjecture of the (symmetric) equilibrium price of fringe sellers $\hat{p}_f$ the markup charged by the granular seller is given implicitly by*

$$\frac{p_l}{w/A_l} = \frac{\sigma+1}{\sigma} + \frac{1}{\sigma}\frac{1-\hat{\theta}}{\hat{\theta}} + \frac{1}{\sigma\hat{\theta}}\left(\frac{1-\hat{\theta}}{2}(p_l/\hat{p}_f)^\sigma + \frac{1+\hat{\theta}}{2}(p_l/\hat{p}_f)^{-\sigma}\right), \tag{2.12}$$

where $\hat{\theta} = (1-m_l)\theta$.

Proof. See Appendix B.3. □

The markup is determined by three components. The first denotes the standard constant CES markup $\frac{\sigma+1}{\sigma}$. The second term captures the additional market power derived from the fact that a customer initially attached to the granular seller effectively only becomes detached with probability $\hat{\theta} \leqslant 1 - m_l$. Due to its granularity the seller will always retain at least a fraction m_l of their initial customer base. This is the additional market power endowed to the large firm by their granularity. Alternatively, note that the first two terms combined correspond to the markup derived from a single sector version of Atkeson & Burstein (2008) under Bertrand competition with elasticity of substitution $\rho = \sigma\theta$ and market shares in terms of

[6] This rules out other, potentially interesting forms of competition. A natural extension in which the granular seller acts as a Stackelberg leader where it takes into account the reaction function of fringe sellers is discussed in Appendix C.2. Others may include price leadership by dominant firms that create focal prices that facilitate price coordination and collusion.

the relative customer base.[7] The intuition is analogous to Atkeson & Burstein (2008). The granular seller operates on a less elastic part of the demand curve as they compete for a residual demand of $1 - m_l$ customers. Thus, the more granular the firm the less it competes for new customers and the more it extracts surplus from the existent customer base.

The final term captures the strategic interaction between firms. It is a weighted average of the relative price and its inverse. It captures two forces. On the one hand, prices are strategic substitutes as higher prices by fringe sellers makes it easier to attract customers through lowering prices. On the other hand, prices are strategic complements as a price rise by fringe sellers allows the granular seller to also increase their price without losing customers. It can be shown formally using the Implicit Function theorem that for sufficiently small $\hat{p}_f/p_l$ prices will be gross substitutes whereas for larger relative prices of fringe sellers the strategies will be gross complements. This captures the following intuition. Suppose that $\hat{p}_f/p_l$ is very small. Then, the granular seller does not find it profitable to compete on such low prices and rather extracts surplus from the existing customer base. If a fringe seller then raises their price it has become easier to attract unattached customers and hence increases the incentive to lower prices to attract them. On the other hand, suppose that $\hat{p}_f/p_l$ is high. Then, the granular seller is already attracting most of the customers. Hence, in response to a rise of the price of fringe sellers it is optimal to follow suit and not attract the few customers remaining. Details on strategic interactions in pricing are delegated to Appendix C.1.

It might be instructive to compare the present model with Bornstein (2021). The author presents a framework in which larger firms charge higher markups, though absent granularity. However, in his paper the exogenous detachment probability θ plays a crucial role. It is required that $\theta < 1$ in order for markups to depend on the customer base. Proposition 1 shows that granularity introduces a dependence between customer base and markups that allows for $\theta = 1$ but replaces that assumption with the requirement that customers consider a finite subset of sellers. Alternatively, we can interpret granularity as a rationale for why the effective $\hat{\theta}$ is less than one.

Fringe Sellers. Given conjectures $\hat{p}_l$ and $\hat{p}_f$ for the equilibrium prices of the granular and the fringe sellers, respectively, a fringe seller's problem is given by

$$
\begin{aligned}
V_j(m(j); m_l) = \max_{p,y} \, & y\left(p - \frac{w}{A_f}\right) \\
\text{s.t.} \quad & y \leqslant \mathcal{M}_j^f(p, \hat{p}_f, \hat{p}_l))\frac{I}{p} \\
& \mathcal{M}_j^f(p, \hat{p}_f, \hat{p}_l) = m(j)\left(1 + m_l\theta\frac{(\hat{p}_l/p)^\sigma - 1}{1 + (\hat{p}_l/p)^\sigma} + (1 - m_l)\theta\frac{1 - (p/\hat{p}_f)^\sigma}{1 + (p/\hat{p}_f)^\sigma}\right).
\end{aligned}
$$

[7] In fact, in a repeated version of the static model there is a direct correspondence between past period's sales share and today's customer base, i.e. $s_{t-1} = \mathcal{M}_{l,t-1} = m_{l,t}$.

We note again that a fringe seller is competing both against the granular seller and other fringe sellers whereas the large seller only competes against the latter.

Proposition 2. *In a symmetric equilibrium in which identical fringe sellers are conjectured to charge the same price p_f and given a conjecture for the the equilibrium price of the large seller $\hat{p}_l$ a fringe seller's markup is given implicitly by*

$$\frac{p_f}{w/A_f} = \frac{\sigma+1}{\sigma} + \frac{1}{\sigma}\frac{1-\theta}{\theta} + \frac{1}{\sigma\theta}\frac{\frac{1-m_l}{1+m_l} + \phi(\hat{p}_l/p_f)^{\sigma} + (1-\phi)(\hat{p}_l/p_f)^{-\sigma}}{1 + \frac{1-m_l}{1+m_l}\frac{1}{2}((\hat{p}_l/p_f)^{\sigma} + (\hat{p}_l/p_f)^{-\sigma})}, \tag{2.13}$$

where $\phi = \frac{1}{2} + \frac{m_l\theta}{1+m_l}$.

Proof. See Appendix B.4. □

The markup is again composed of three components. The first is the standard CES constant markup. The second term captures the additional market power endowed to fringe sellers from search frictions. In particular, it is increasing in the fraction $1-\theta$ of initially attached customers that do not get to search for an alternative seller. However, note that relative to the granular seller the advantage is smaller as fringe sellers are infinitesimal. In contrast to the granular seller, markups of fringe sellers are independent of their customer base. Alternatively, note that the first two terms correspond to the markup of a model of monopolistic competition with CES demand and an elasticity $\rho = \sigma\theta$.

The third term again captures the strategic interactions between fringe sellers and the granular seller. The intuition is similar to the final term in (2.12), though the forces are more muted. Intuitively, as $m_l \to 1$ strategic substitutability and complementarities approach that of the granular seller in (2.12), mutatis mutandis (replacing $\hat{\theta}$ with θ). On the other hand, as $m_l \to 0$ the markup of fringe sellers given by (2.13) will be independent of the granular seller and strategic interactions vanish. So intuitively, for intermediate values of m_l, the strength of these strategic forces will be biased towards zero.

2.4 Equilibrium

Definition 1. *An equilibrium to the the game between granular and fringe firms is a price vectors (p_l, p_f) which solves the optimality conditions (2.12) and (2.13) such that each conjecture is correct, $p_l = \hat{p}_l$ and $p_f = \hat{p}_f$.*

Proposition 3. *Suppose that $\sigma < 1$.*

1. *If $\underline{\Gamma}(m_l, \theta) \leqslant \frac{A_f}{A_l} \leqslant 1$, where $\underline{\Gamma}(m_l, \theta) = \left(\frac{\sqrt{1-m_l\theta^2 - \frac{1}{2}\theta(1-m_l)}}{1+\frac{1}{2}(1+m_l)\theta}\right)^{1/\sigma} < 1$, then there exists a unique equilibrium in which the granular seller charges a higher markup, i.e. $\frac{p_l}{w/A_l} > \frac{p_f}{w/A_f}$.*
2. *If $A_f > A_l$, there exists a threshold $\underline{m}(\theta) < 1$ such that there exists a unique equilibrium in which a granular seller with initial customer base $m_l > \underline{m}(\theta)$ charges a markup above that of fringe sellers. A sufficient condition*

defines the threshold $\underline{m}(\theta)$ *as*

$$\underline{m}(\theta) = \inf\left\{m \in \mathbb{R}^+ : \Phi(m;\theta) \geqslant 0\right\}, \tag{2.14}$$

where $\Phi(m;\theta) = \inf_{x\geqslant 1}\left\{\left(\frac{3m-1}{1-m} + \frac{1}{2}(x+\frac{1}{x})\right)\left(1 + \frac{1+(1-m)\theta}{2}\frac{1}{x} + \frac{1-(1-m)\theta}{2}x\right) - \theta(x - \frac{1}{x})\right\}$*. The threshold further satisfies* $\underline{m}(\theta) < \frac{-(1-\theta)+\sqrt{1-2\theta+4\theta^2}}{3\theta} \leqslant 1/\sqrt{3}$ *for all* $0 < \theta \leqslant 1$.

3. *Let* $A_l = A_f$*, then* $\lim_{m_l \to 0} \frac{p_l}{p_f} = 1$*. For any* $0 < A_l/A_f < \infty$, $\lim_{m_l\to 1} \frac{p_l}{p_f} = \infty$.

Proof. See Appendix B.5. □

Point 1. formally establishes the result that granularity endows firms with additional market power, the intuition of which was discussed above. Even if fringe and granular sellers are identical in terms of productivity the large seller charges a higher markup. The bound $\underline{\Gamma}(m_l, \theta)$ is the required sufficient condition to ensure equilibrium uniqueness. The parameter assumption that $\sigma < 1$ is crucial for the best response correspondence to be single-valued and is maintained throughout the paper.[8]

Point 2. extends the result to the case in which fringe sellers have the productivity advantage. In the static model, more productive sellers tend to charge higher markups (see Appendix C.1). The Proposition states that if $A_l < A_f$ there exists a threshold $\underline{m}(\theta)$ for each parameter value θ such that if the large seller is sufficiently granular, i.e. $m_l \geqslant \underline{m}(\theta)$, it charges a higher markup than the fringe sellers even if $A_l < A_f$.

Point 3. captures two facts. First, as $m_l \to 0$ the granularity of the large seller vanishes and they behave as a fringe seller of the same productivity. This confirms that the only additional source of market power for the large seller is its granularity. Second, as $m_l \to 1$ the large seller effectively faces no competition and charges the monopoly price which is infinite in this setting.

Best response functions and optimal prices as a function of the granular seller's customer base are plotted in Figure 3 for the case where $A_l = A_f$. First, we note that the granular seller indeed charges a higher price and markup. Second, the price of the large seller is increasing in the granular seller's customer base and converges to that of the fringe seller's price of the same productivity as $m_l \to 0$. This is robust across different specifications. Whether the price of the fringe seller is increasing or decreasing in the granular seller's customer base strongly depends on the relative productivity. If fringe sellers are relatively less productive prices are strategic substitutes. If they are relatively more productive prices are strategic complements. In the strategic complements case, the presence of a granular firm has a purely price-increasing effect. A crucial assumption is that customers choose between a finite subset of sellers. Search frictions may then induce *price-increasing competition* where the introduction of another seller raises all prices in the economy.[9]

[8]This is the same restriction required to ensure equilibrium uniqueness in the static model of Bornstein (2021).

[9]Chen & Riordan (2008) show that price-increasing competition is a feature of discrete choice model of production differentiation and an outside option. A duopoly equilibrium may feature higher prices than the corresponding monopoly outcome as firms trade off price-reducing competition for market share with a lower price-sensitivity of their average customers.

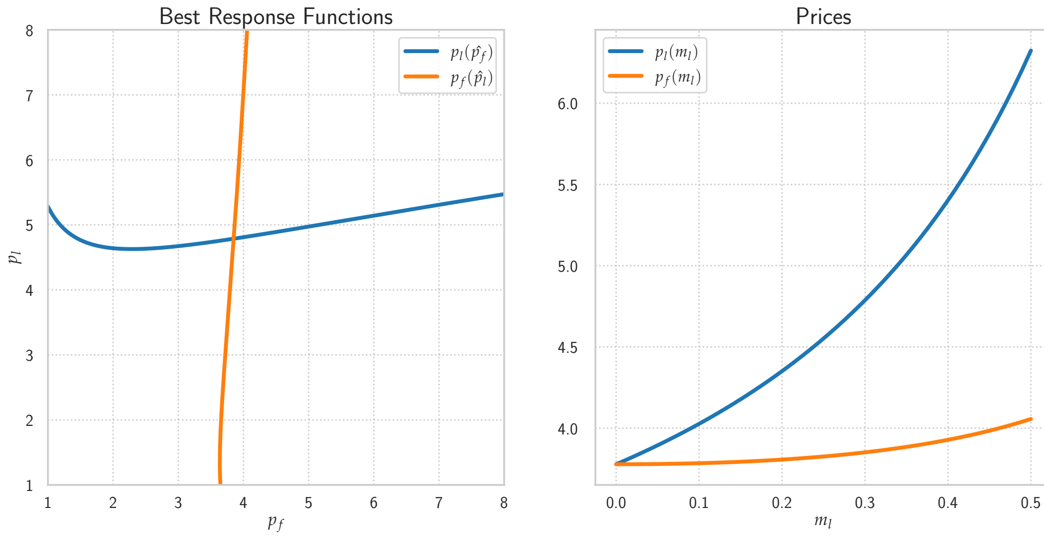

Figure 3: Best Response Functions in Static Game.

Parameter values: $A_l = A_f = 1$, $w = 1$, $\sigma = 0.9$, $\theta = 0.8$, $m_l = 0.3$ (left panel).

3 Dynamic Model

This section introduces a straightforward dynamic extension of the static model. Time is discrete. Factor input prices are constant, the granular seller has constant productivity and fringe sellers are allowed to differ in terms of their productivity which evolves stochastically over time. The extension is motivated by the observation that customer acquisition motives are inherently intertemporal. There is a central trade-off between reduced profits today and returns on the customer capital tomorrow. Further, important questions such as how the granularity of a single seller affects firm dynamism of fringe sellers necessitate a dynamic framework. This section attempts to address these points.

3.1 Environment

Demand. Households are myopic and do not internalise the intertemporal implications of their decisions. Households behave exactly as in the static model. In particular, they do not take into account that they may get locked in after a purchase. Each household inelastically supplies one unit of labour. Aggregate firm profits Π are distributed uniformly and lump-sum. The income of households therefore is

$$I = w + \Pi. \tag{3.1}$$

Supply. I consider a one-sector economy.[10] As in the static model, there is one large granular seller indexed by l and a continuum of fringe sellers indexed by $j \in [0, J]$. The granular seller has constant productivity while the productivity of fringe sellers follows a stationary AR(1) process, $\log a_{f,t+1} = \rho_a \log a_{f,t} + \sigma_a \nu_{t+1}$, where $\nu_{t+1} \sim \mathcal{N}(0,1)$ is drawn iid across firms and periods.[11] There is no entry and exit of firms and no transitions between fringe firms and the granular seller.

Search frictions are analogous to the static model. The timing is as follows. Search occurs in the beginning of the period and sales at the end. The beginning of $t+1$ customer base of initially attached customers equals the end of period t customer base.

Throughout, I assume that the strategies of firms are only functions of the pay-off relevant state variables.[12] The granular firm has one state variable, its beginning of period customer base m_l. Fringe seller j has three state variables, its beginning of period customer base $m(j)$, its current productivity $a(j)$ and the beginning of period customer base m_l. As shown in Appendix A.2, the assumption on the matching process ensures that for the purpose of computing policy functions the state space can effectively be reduced to $(a(j), m_l)$. Note that the state space of the fringe seller includes the customer base of the granular seller. This is because the state variables of a granular seller enter as aggregate state variables as is well known from the granularity literature.

There are two main differences relative to the simple static model. First, the continuation value of the customer base captures an intertemporal investment motive for firms. Second, the heterogeneity of productivity induces a distribution of fringe sellers. Let the joint density of fringe sellers over their customer base and productivity be denoted by $\Lambda(a, m)$ for a given customer base of the large seller m_l. Then, the meeting rate with fringe sellers of productivity a is given by $\int m\Lambda(a, m)dm = (1 - m_l)\lambda(a)$ where

$$\lambda(a) = \frac{1}{1 - m_l} \int m\Lambda(a, m)dm, \tag{3.2}$$

$$= \frac{\mathbb{E}_J[m|a]}{\mathbb{E}_J[m]} \pi(a), \tag{3.3}$$

where expectations are taken over fringe sellers $j \in [0, J]$ and π denotes the cross-sectional productivity distribution. The decomposition of the meeting rate is intuitive. With probability $1 - m_l$ a customer matches with one of the fringe sellers. Conditional on a match with a fringe seller the customer meets a firm of productivity a at the rate $\lambda(a)$. Equation (3.3) derives a simple expression for λ that has a straightforward interpretation. If search was uniform across fringe sellers $j \in [0, J]$ the meeting rate with a fringe seller of productivity a would be $\pi(a)$. But as meeting rates are stochastically increasing in the

[10] Alternatively, suppose the economy is endowed with a unit measure of sectors $s \in [0, 1]$ with Cobb-Douglas utility over sectors and nesting (2.1) as a subutility for each sector, i.e. $U(\boldsymbol{C}) = \int_0^1 \log \left(\sum_{k \in \mathcal{C}_s} \exp\left(\sigma^{-1}\epsilon_{ks}\right) c_{ks}\right) ds$ where $\mathcal{C}_s$ denotes the choice set in each sector s and ϵ_{ks} a Gumbel taste shock iid over households, products and sectors. Households allocate their expenditure uniformly across sectors, yielding a formulation for each sector identical to the one above (with no additional insight).

[11] The assumption of constant productivity on the granular seller ensures that a non-stochastic stationary equilibrium exists.

[12] This assumption rules out implicit collusive agreements between firms.

customer base we scale π by an adjustment factor capturing the relative mass of customers attached to a fringe seller of productivity a versus any fringe seller.

3.2 Granular Seller

For any conjecture of the policy function of fringe sellers $\hat{p}_f(a, m_l)$ the granular seller's problem is

$$
\begin{aligned}
V_l(m_l) = \max_{p,y,m_l'} & \; y\left(p - \frac{w}{a_l}\right) + \beta V_l(m_l') \\
\text{s.t. } & y \leqslant \mathcal{M}_l(p, \hat{p}_f)\frac{I}{p} \\
& \mathcal{M}_l(p, \hat{p}_f) = m_l \left(1 + (1 - m_l)\theta \int_0^\infty \frac{1 - (p/\hat{p}_f(a, m_l))^\sigma}{1 + (p/\hat{p}_f(a, m_l))^\sigma}\lambda(a)da\right) \\
& m_l' = \mathcal{M}_l(p, \hat{p}_f)
\end{aligned}
\tag{3.4}
$$

where the objective is to maximise current profits plus the continuation value of next period's customer base. The first constraint captures that sales are constrained by both the size of the customer base and the household demand per purchase. The second constraint defines the law of motion for the end of period customer base. The third constraint captures the evolution of the customer base of the large seller over time.

3.3 Fringe Sellers

For any conjecture of the policy functions of the granular seller $\hat{p}_l(m_l)$ and the fringe sellers $\hat{p}_f(a, m_l)$ the problem of fringe seller j is

$$
\begin{aligned}
V_j(a, m_l) = & \max_p \Delta_j^f(p, \hat{p}_f, \hat{p}_l)\left[(p - \frac{w}{a})\frac{I}{p} + \beta \int V_j(a', m_l')P(da'|a)\right] \\
\text{s.t. } & \Delta_j^f(p, \hat{p}_f, \hat{p}_l) = 1 + \theta m_l \frac{1 - (p/\hat{p}_l(m_l))^\sigma}{1 + (p/\hat{p}_l(m_l))^\sigma} + \theta(1 - m_l)\int_0^\infty \frac{1 - (p/\hat{p}_f(\tilde{a}, m_l))^\sigma}{1 + (p/\hat{p}_f(\tilde{a}, m_l))^\sigma}\lambda(\tilde{a})d\tilde{a}
\end{aligned}
\tag{3.5}
$$

as well as subject to the Law of Motion for the customer base of the large seller defined above. Most importantly, the value function and hence the pricing of fringe sellers is independent of their initial customer base. This results from the specific product market search technology assumed and is discussed in Paciello et al. (2019).[13] There are two countervailing forces. On the one hand, a higher $m(j)$ raises incentives to extract surplus from the $(1 - \theta)m(j)$ captured customers. On the other hand, a higher $m(j)$ increases encounters with new prospective customers. In the formulation above, these two forces cancel exactly and prices of fringe sellers are independent of $m(j)$.

[13] In particular, the matching technology implies that outflows and inflows conditional on a match scale equally with size. Further, there are no fixed costs and the production technology is CRS. Each contributes to the profit function to scale linearly in the initial customer base.

3.4 Stationary Markov-Perfect Equilibrium

Definition 2. *A stationary Markov-perfect equilibrium is a set of policy functions $p_l(m_l)$ and $p_f(a, m_l)$, value functions $V_l(m_l)$ and $V_j(a, m_l)$, a customer base $m_{l,ss}$ of the large seller and a distribution of customers over marginal costs $\lambda(a)$ such that*

1. *Consumption policy functions solve the household problem (2.1) subject to the income as defined by (3.1).*
2. *Pricing policy functions of the granular and fringe sellers solve (3.4) and (3.5), respectively, and conjectures are correct. That is $p_l(m_l) = \hat{p}_l(m_l)$ for all $m_l \in [0, 1]$ and $p_f(a, m_l) = \hat{p}_f(a, m_l)$ for all $(a, m_l) \in \mathbb{R}^+ \times [0, 1]$.*
3. *The customer base of the granular seller is constant, i.e. $m_l' = m_l = m_{l,ss}$, and the measure λ is constant. This jointly implies*
$$p_l(m_{l,ss}) \geqslant \left[\int_0^\infty \frac{2}{p_l(m_{l,ss})^{-\sigma} + p_f(a, m_{l,ss})^{-\sigma}} \lambda(a) da \right]^{1/\sigma}, \tag{3.6}$$
which holds with equality if $m_{l,ss} > 0$. Additionally, for all Borel sets $\mathscr{A} \subset \mathbb{R}^+$
$$\lambda(\mathscr{A}) = \int_0^\infty \Delta_j^f(p_f(\tilde{a}, m_{l,ss}), p_f(\cdot, m_{l,ss}), p_l(m_{l,ss}))\lambda(\tilde{a}) P(\mathscr{A}|\tilde{a}) d\tilde{a}. \tag{3.7}$$

It is immediate from the law of motion of the customer base in (3.4) that $m_{l,ss} = 0$ is always a steady state. For the remainder of the paper we focus, whenever it exists, on the stationary equilibrium that features $m_{l,ss} > 0$.

Technically, I use a slightly modified stationary Markov-Perfect equilibrium concept. That is because the granular seller should in principle also take into account the evolution of the distribution $\lambda' = \Gamma(\lambda; \boldsymbol{s})$ where $\boldsymbol{s} = (p_l, \hat{p}_f, m_l)$. This is true even in a stationary equilibrium, as the pricing decision of the large seller, due to its granularity, directly changes the law of motion for the measure λ'. The large seller might then find it optimal to internalise its ability to change the evolution of λ. For example, by marginally lowering its price the granular seller can steal more customers from some productivity types than others.[14] Unless productivity is iid over time this will translate into changes to the evolution of the density λ'. Away from this special case, the large seller should then internalise its effect on λ', which I abstract from in the problem of the granular seller (3.4).[15]

To keep the model tractable, I therefore compute an approximation to the full solution in the spirit of the

[14] By marginally lowering its price the granular seller can steal more customers from fringe sellers of productivity types that charge prices close to the large seller or that have a higher density $\lambda(a)$. As an illustration, consider the discretisation of the productivity process into three levels (A_1, A_2, A_3). Now consider a price p_l such that $p_f(A_1) > p_l > p_f(A_2) > p_f(A_3)$. Then, A_2 and A_3 see an increase in the net flow into their customer base. This will reallocate mass to $\lambda(A_2 \cup A_3)'$ and away from $\lambda(A_1)'$. If instead $p_f(A_1) > p_f(A_2) > p_l > p_f(A_3)$, then only productivity level A_3 will see an increase in the net flow into their customer base. This will reallocate mass to $\lambda(A_3)'$ and away from $\lambda(A_1 \cup A_2)'$. Hence, the granular seller could use its pricing to reshuffle mass between productivity levels.

[15] If the large seller internalises the effect of its pricing on the measure λ' a full solution should take this into account, even if the measure λ is constant.

partially oblivious equilibrium of Ifrach & Weintraub (2016).[16] The solution concept can be viewed as a numerical approximation or a behavioural model in its own right. It is supposed that firms know their own idiosyncratic states and those of the large seller. However, firms do not attend to the states of fringe sellers except their own. Instead, their beliefs about the states of fringe sellers equals their long-run state. This implies that while all firms track changes to the aggregate mass of customers attached to fringe sellers they form their belief about fringe sellers according to $\lambda = \lim_{T\to\infty} \tilde{\Gamma}(\lambda_T, s_T)$ where $\tilde{\Gamma}(\cdot)$ denotes the transition function mapping current distribution λ into next period's distribution λ' under the partially oblivious strategies. That is, all agents take the measure λ as given. In Appendix C.3, I argue that this approximates the full solution reasonably well.

3.5 An Analytical Example

Suppose that the productivity process for fringe sellers is degenerate and there is a single type A_f of fringe sellers as in the static case before, i.e. $\sigma_a = 0$. Denote the corresponding productivity level of the granular seller by A_l. This renders the set-up sufficiently tractable to characterise the steady state analytically.

Proposition 4. *If $A_l > A_f$, then the steady state in which fringe sellers play symmetric strategies is characterised by*

1. *All firms charge identical prices in steady state*

$$p_l(m_{l,ss}) = p_f(m_{l,ss}), \tag{3.8}$$

   implying that relative markups are

$$\mu_l/\mu_f = A_l/A_f > 1. \tag{3.9}$$

2. *The steady state granularity of the large seller is endogenously determined as*

$$\frac{m_{l,ss}}{1-m_{l,ss}} = (A_l/A_f - 1)\left[1 + \frac{\sigma\theta}{2(1-\beta)}\right]. \tag{3.10}$$

3. *Markups are*

$$\mu_f = 1 + \frac{2(1-\beta)}{\sigma\theta}, \tag{3.11}$$

$$\mu_l = \mu_f + \frac{2(1-\beta)}{\sigma\theta}\frac{m_{l,ss}}{1-m_{l,ss}} = \left(1 + \frac{2(1-\beta)}{\sigma\theta}\right)\frac{A_l}{A_f}. \tag{3.12}$$

If $A_l \leqslant A_f$, there does not exist a non-degenerate steady state.

[16] The solution concepts of Ifrach & Weintraub (2016) and relatedly Benkard et al. (2015) are used in the IO literature to tackle similar problems featuring a few dominant and many small firms.

Proof. See Appendix B.6. □

Proposition 4 highlights an important insight from the model. A firm can only remain granular in steady state if it is sufficiently productive.[17] This shows that long-run forces discipline the market power derived from granularity. It is confirmed numerically that when $A_l \leqslant A_f$ the large seller's granularity will vanish in the long-run. Granularity is not an exogenous feature of the economy, but derives from the productivity gap between large and fringe sellers as seen from (3.10).

Point 1. of Proposition 4 establishes that in steady state the large seller and fringe sellers must charge the same price. Intuitively, if the large seller was charging a price above that of fringe sellers, i.e. $p_l > p_f$, then it would see its customer base fall to zero in the long-run. This cannot be consistent with profit maximisation if the large seller is sufficiently productive as they would always prefer to lower the price to attract some customers. On the other hand, if $p_l < p_f$ then the large seller captures the entire market in the long-run – effectively turning the seller into a monopolist. In the present model, a monopolist charges a price diverging to infinity. Hence, $p_l < p_f$ in the long-run is also not consistent with profit maximisation. It can then be shown formally via proof by exhaustion that $p_l(m_{l,ss}) = p_f(m_{l,ss})$ must be the steady state for $A_l > A_f$. If $A_l \leqslant A_f$, then the large seller finds it less profitable to keep prices low and maintain a positive customer base in the long-run than to extract greater surplus from existent customers today and vanish in the long-run.

Point 3. of Proposition 4 highlights another important feature of the model which we will return to in section 4. Equation (3.11) shows that the markup of fringe sellers is given exogenously by the primitives of the economy and does not depend on the granular seller. This is a consequence of the equilibrium outcome (3.8) that prices are equalised. On the other hand, equation (3.12) shows that the markup of the large seller is directly linked to fringe sellers through their productivity lead. Equation (3.12) also sheds light on the adjustment mechanism. The productivity gap feeds into the steady state customer base of large sellers which shape incentives for the optimal markups.

Corollary 1. *The steady state customer base of the granular seller is increasing in its relative productivity A_l/A_f, the discount factor β and the detachment probability θ. The steady state markup of the granular seller is increasing in relative productivity A_l/A_f, but decreasing in the discount factor β and the detachment probability θ.*

Proof. See Appendix B.7. □

Corollary 1 captures the tension between two forces in the model. On the one hand, the granular seller wishes to build up its customer base as its granularity endows them with additional market power. On the other hand, the only mechanism with which to attract new customers in the model is by lowering prices. This puts discipline on the pricing by the granular seller.

[17] While Proposition 4 does not prove convergence to a steady state, this has been found numerically to be a robust feature of the model for any non-degenerate initial customer base of the large seller.

3.6 Quantitative Model

Let us turn to the full quantitative model with stochastic productivity of fringe firms. To clarify notation, denote by $\mathbb{E}_\lambda[X] = \int X(a)\lambda(da)$ and $\mathbb{V}ar_\lambda[X] = \int (X(a) - \mathbb{E}_\lambda[X])^2 \lambda(da)$ the expectation and variance taken with respect to the measure λ as defined in (3.3). Dynamic markups are characterised in Proposition 5.

Proposition 5. *The Proposition summarises key results about the dynamic markup. Function arguments are suppressed for notational convenience.*

1. *Under suitable boundedness conditions the optimal sequence of markups for the granular seller $\{\mu_{l,t+\tau}\}_{\tau \geqslant 0}$ is given by*

$$\mu_{l,t} = \frac{1 + \varepsilon_{\mathcal{M}_{l,t}}}{\varepsilon_{\mathcal{M}_{l,t}}} - \sum_{\tau=1}^{\infty} \beta^\tau \mathbb{E}_t \left[\left(\prod_{k=1}^{\tau} \frac{\partial \mathcal{M}_{l,t+k}}{\partial m_{l,t+k}} \right) \frac{\mu_{l,t}}{\mu_{l,t+\tau}} (\mu_{l,t+\tau} - 1) \frac{I_{t+\tau}}{I_t} \right], \tag{3.13}$$

   where $\varepsilon_{\mathcal{M}_l} = -\frac{\partial \mathcal{M}_l}{\partial p_l} \frac{p_l}{\mathcal{M}_l}$ and denotes the markup function in absence of dynamics.

2. *In the stationary equilibrium the markup reads*

$$\mu_l = (1-\beta)\frac{1 + \varepsilon_{\mathcal{M}_l}}{\varepsilon_{\mathcal{M}_l}} + \beta \tag{3.14}$$

$$= 1 + \frac{1-\beta}{2\sigma\theta(1 - m_{l,ss})} \left[1/4 + \mathbb{V}ar_\lambda \left(\frac{1}{1 + (p_l/p_f)^\sigma} \right) \right]^{-1}. \tag{3.15}$$

3. *The dynamic markup for a fringe seller j with current productivity a_t is given by*

$$\mu_{j,t} = \frac{\varepsilon_{\Delta,t} + 1}{\varepsilon_{\Delta,t}} - \sum_{\tau=1}^{\infty} \beta^\tau \mathbb{E} \left[\left(\prod_{k=1}^{\tau} \Delta_{j,t+k} \right) \frac{\mu_{j,t}}{\mu_{j,t+\tau}} (\mu_{j,t+\tau} - 1) \frac{I_{t+\tau}}{I_t} | a_t \right], \tag{3.16}$$

   where $\frac{\varepsilon_\Delta + 1}{\varepsilon_\Delta} = -\frac{\partial \Delta}{\partial p_f} \frac{p_f}{\Delta}$ and denotes the markup function in absence of dynamics.

Proof. See Appendix B.8. □

Proposition 5 captures the customer investment motive of firms. The dynamic markup function equals the static counterpart minus the discounted stream of future marginal values of higher customer capital. The marginal value is greater if (i) markups are expected to rise in the future, (ii) aggregate demand is expected to grow, and (iii) a rise in next period's customer base has persistent effects on future demand.

In particular, the third effect implies that if a firm anticipates that adding a new customer will only have transitory effects for future demand, i.e. small $\partial \mathcal{M}_{l,t+k} / \partial m_{l,t+k}$ and $\Delta_{j,t+k}$, it finds it optimal to extract profits from their current customer base and charge higher markups. As a case in point, low productivity fringe sellers charge higher prices as they find it too costly to keep their prices as low as their high productivity competitors. In equilibrium, they thus expect any increase in their customer base to only have transitory effects on future demand. This lowers the continuation value of their customer capital, which lowers incentives to attract new customers by reducing prices. As a result, if intertemporal motives receive a sufficiently large weight, i.e. β sufficiently large, then markups of fringe sellers are decreasing in productivity. Hence, more productive fringe sellers invest more to grow due to their higher continuation value and they do so by charging lower markups.

In contrast, note that for the granular seller markups are increasing in their productivity as was already noted in the limiting case of Proposition 4. A higher productivity level allows the large seller to sustain a greater level of granularity. This endows the large seller with additional market power which it uses to charge higher markups – a mechanism absent for fringe sellers.

The role of the discount factor β is familiar from the literature. It modulates the intertemporal investment motive. The greater β the more firms value future profits and the lower today's prices in order to invest in a greater customer base. This is illustrated most clearly for the granular seller in equation (3.14). The dynamic markup for the granular seller in a stationary equilibrium is a weighted average of the static markup and the competitive markup where weights correspond to the strength of the intertemporal motive β.

Numerical Example. To illustrate the workings of the model we consider the following numerical example. In the benchmark, we set $\theta = 0.4$, $\sigma = 0.99$, $\beta = 0.9$, $\rho_a = 0.95$, $\sigma_a = 0.05$, and $A_l = 1.3$.[18] While a careful calibration is beyond the scope of the paper and this section serves as an illustrative numerical example, the benchmark parametrisation matches the aggregate sales-weighted markup in De Loecker et al. (2020) of 1.6 and a market concentration of the large seller of $0.25 - 0.3$. The latter is in line with the sales share of the four largest firms in an industry in Autor et al. (2020), averaged across sectors. To be consistent with the data, I view the single large seller in the model as representative of the four largest firms.

In the model, it is natural to weight by the customer share of firms which here is equivalent to weighting by the sales share.[19] Weighting by the customer share also underlies the derivation of the relevant measure λ from (3.3). Table 1 summarises the results. On average, the granular seller charges higher markups, i.e. $\mu_l > \mathbb{E}_\lambda \mu_f$. The above discussed properties of dynamic markups are confirmed numerically. A higher productivity of the large seller allows them to raise their markup and enjoy a greater granularity. The results also stress the importance of the discount factor for moderating markups. Further, the calibration features reallocation of customers from unproductive to more productive fringe sellers as seen by the positive Kullback-Leibler divergence $D_{KL}(\pi|\lambda)$. If customers were uniformly allocated across fringe sellers then $\lambda = \pi$ and the divergence measure would be zero. The greater $D_{KL}(\pi||\lambda)$ the stronger the reallocation of customers from the uniform baseline.

[18]The restriction $\sigma < 1$ is required for equilibrium uniqueness. As this roughly corresponds to a CES demand elasticity of $1 + \sigma$ this might be counter-factually too small. However, introducing imperfectly substitutable outside goods from other industries could allow for empirically more realistic σ.

[19]To see this note that $s_{i,t-1} = \frac{p_{i,t-1} y_{i,t-1}}{p_{l,t-1} y_{l,t-1} + \int p_{t-1}(j) y_{t-1}(j) dj} = \frac{\mathcal{M}_{i,t-1}}{\mathcal{M}_{l,t-1} + \int \mathcal{M}_{t-1}(j) dj} = m_{i,t}$ as $y_{i,t-1} = \mathcal{M}_{i,t-1} \frac{I}{p_{i,t-1}}$.

	μ_l	$\mathbb{E}_\lambda \mu_f$	$\mathbb{E}\mu_i$	$m_{l,ss}$	$\frac{p_l}{(\mathbb{E}p_f^\sigma)^{1/\sigma}}$	$\mathbb{V}ar_\lambda(p_f^\sigma)$	$D_{KL}(\pi\|\|\lambda)$
Baseline	1.73	1.55	1.59	0.27	0.97	0.13	0.61
$A_l = 1.5$	1.99	1.54	1.74	0.43	0.97	0.13	0.62
$\sigma_a = 0.01$	1.95	1.51	1.69	0.42	1.0	0.01	0.03
$\beta = 0.7$	2.98	2.54	2.64	0.23	0.98	0.18	0.31

Table 1: Summary Statistics of Stationary Equilibrium

Note: $D_{KL}(\pi||\lambda)$: Kullback-Leibler divergence between λ and π.

Figure 4 plots the stationary equilibrium. The left panel shows the relative markup of the granular seller to the fringe sellers, μ_l/μ_f, as a function of relative productivity of fringe sellers to the granular seller. The granular seller charges 20% higher markups than a fringe seller of identical productivity and it is only the most unproductive fringe sellers that charge a higher markup. The middle panel illustrates the reallocation of customers across productivity types. In particular, it plots the ratio $\frac{\lambda(a)}{\pi(a)} = \frac{\mathbb{E}_J[m|a]}{\mathbb{E}_J[m]}$ where π denotes the stationary, cross-sectional productivity distribution.[20] If $m \perp a$, the ratio would be $\mathbb{E}_J[m|a]/\mathbb{E}_J[m] = 1$ for all a. The fact that $\frac{\mathbb{E}_J[m|a]}{\mathbb{E}_J[m]}$ is increasing in a implies that customers are reallocated from low to high productivity firms through optimal search by households. The right panel plots the evolution of the customer bases when productivity is permanently high, medium or low. Highly productive firms grow quickly while low productivity firms vanish over time.

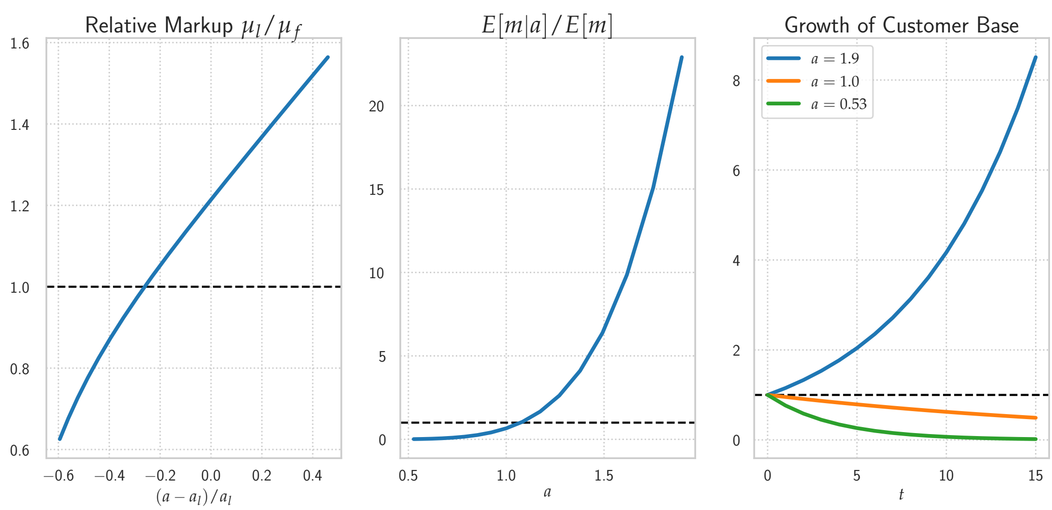

Figure 4: Stationary Equilibrium.

[20]Note that the ratio is intimately related to the Kullback-Leibler divergence $D_{KL}(\pi||\lambda) = -\int \pi(a) \log_2 \left(\frac{\mathbb{E}_J[m|a]}{\mathbb{E}_J[m]}\right) da$.

4 A Neutrality Result

In this section, I introduce an approximate neutrality result. I will show that for a reasonable matching process as outlined above the presence of a granular seller is neither relevant for the pricing decisions of fringe sellers nor for their growth.[21] However, the presence of a granular seller will matter for industry profits and the market share of fringe sellers. That is, policy functions are neutral but the scale is not. To this end, I compare the stationary equilibria of two economies, one without and the other with a granular seller, and find that policy functions and firm growth are indistinguishable between the two economies. The output of the respective models is plotted in Figure 5.

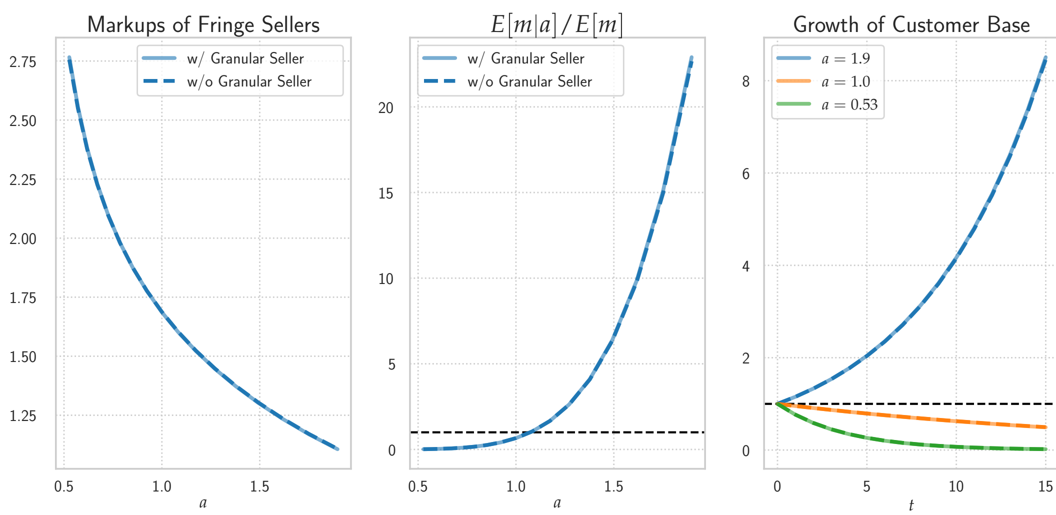

Figure 5: Neutrality under Presence of Granular Seller.

4.1 Mechanism

Intuitively, in the stationary equilibrium the granular seller behaves as if it was an aggregate version of all fringe sellers. As the granular seller is *mimicking* the behaviour of an aggregation of fringe sellers this is as if a fringe seller is only competing against the aggregate of other fringe sellers. It is then almost immediate that the presence of a granular seller is irrelevant.

The main intuition has already been presented in Proposition 4 in which it was shown for the case in which $\sigma_a = 0$ that the markup for fringe sellers is determined by the primitives of the economy while the markup of the granular seller adjusts to solve the fixed point $\mathcal{M}_l(p_l(m_{l,ss}), p_f(\cdot, m_{l,ss}); m_{l,ss}) = m_{l,ss}$. As such equilibrium markups of fringe sellers are independent of the granular seller. Instead, the customer base of the granular seller is the variable that adjusts to ensure equilibrium.[22]

[21] A similar approximate neutrality result is derived in Wang & Werning (2020) who show that *quantitatively* a standard Phillips curve provides an excellent approximation to an exact Phillips curve featuring oligopolistic sectors.

[22] This is akin to standard firm dynamics models with free entry in which the mass of entrants does most of the adjusting. For

To understand this in the general case, first note that a sufficient condition for an exact neutrality is that the Law of Motion of the fringe seller's customer base is identical in the stationary equilibrium with and without a granular seller. That is, $\Delta_j^f(p, \hat{p}_f, \hat{p}_l) = \Delta_{j,-l}^f(p, \hat{p}_f)$ for all $p > 0$ and $\hat{p}_f$ corresponding to the equilibrium policy function of fringe sellers in an economy without a granular seller. From (3.5) it is then immediate that policy functions are neutral under the presence of a large seller.

While exact neutrality does not hold we can show that an approximate version applies if the price dispersion is not too large. First, condition (3.6) for a stationary equilibrium requires that the price of the large seller is some expected harmonic average of all possible competition scenarios against fringe sellers. The customer base of the large seller adjust such that its optimal price satisfies $p_l^\sigma = \mathbb{E}_\lambda\left[\frac{2}{p_l^{-\sigma}+p_f^{-\sigma}}\right]$, suppressing function arguments for notational convenience. This formalises the notion that the granular seller acts as an aggregation of fringe sellers. Using this equilibrium condition we can derive the following Lemma.

Lemma 3. *Denote the Law of Motion for the customer base of fringe sellers by Δ_j^f and $\Delta_{j,-l}^f$ when a granular seller is present and absent, respectively. Then, in a stationary equilibrium up to a second order approximation and ignoring higher order terms*

$$\sup_{p>0} |\Delta_j^f(p, \bar{p}_f, p_l) - \Delta_{j,-l}^f(p, \bar{p}_f)| = \sup_{p>0} \; m_{l,ss}\theta \frac{p^\sigma |\mathbb{E}_\lambda \bar{p}_f^\sigma - p^\sigma|}{\mathbb{E}_\lambda \bar{p}_f^\sigma (\mathbb{E}_\lambda \bar{p}_f^\sigma + p^\sigma)^3} \mathbb{V}ar_\lambda(\bar{p}_f^\sigma) \tag{4.1}$$

$$\leqslant m_{l,ss}\theta \frac{\mathbb{V}ar_\lambda(\bar{p}_f^\sigma)}{[\mathbb{E}_\lambda \bar{p}_f^\sigma]^2}, \tag{4.2}$$

where $\bar{p}_f$ corresponds to the equilibrium policy function of fringe sellers in an economy without a granular firm.

Proof. See Appendix B.9. □

The above Lemma shows that approximate neutrality holds when the granular seller is not too large or price dispersion weighted by λ is small relative to the average price.[23] Numerical simulations verify this intuition. Further, it is immediate that if the productivity process is degenerate as in the analytical example then $\bar{p}_f^\sigma$ is a constant and exact neutrality holds.

We note that the approximate neutrality result did not depend on the specific form of competition assumed. The steady state condition for the customer base of the large seller together with the assumptions on the matching process ensured approximate neutrality. Thus, the result applies more widely, e.g. in the case of Stackelberg competition (see Appendix C.2).

What Neutrality Does Not Mean. While we have shown that the behaviour of fringe firms is approximately neutral under the presence of a granular seller it is also instructive to stress what the neutrality result

example, in a simple variant of the Hopenhayn-Rogerson model a change to labour supply is fully absorbed by a change in the mass of entrants, leaving unchanged other variables such as the wage.

[23] As customers concentrate at the most productive firms and productivity is persistent, λ is concentrated. Hence, price dispersion is not too large as very low or very high prices receive a small weighting.

does not say. It does not claim that fringe sellers are as well off in an economy with as in an economy without a granular seller. In fact, the average value of fringe seller of productivity a is lower in an economy with a large seller since

$$\begin{aligned}\mathbb{E}\left[mV_j(a, m_{l,ss})|a\right] &= \mathbb{E}\left[m|a\right] V_j(a, m_{l,ss}) \\ &= (1 - m_{l,ss})\frac{\lambda(a)}{\pi(a)} V_j(a, m_{l,ss}) \\ &\leqslant \frac{\lambda(a)}{\pi(a)} V_j(a, m_{l,ss}) = \frac{\lambda(a)}{\pi(a)} V_{j,-l}(a),\end{aligned}$$

which holds with strict inequality as long as $m_{l,ss} > 0$. The third line establishes the inequality between the average value in the presence and absence of a granular seller when approximate neutrality holds, i.e. $V_j(a, m_{l,ss}) = V_{j,-l}(a)$. Thus, while policy functions are neutral the same does not hold for their customer base and profits.

Two integral assumptions to obtain the approximate neutrality result are the abstraction from entry and exit and that fringe sellers' value functions are homogeneous of degree 1 in their customer base. The latter implies that policy functions are independent of the customer base for fringe sellers. So a loss of customers to the granular firm has no effect on the behaviour of fringe sellers. The former ensures that the value function is in fact homogeneous of degree 1. Introducing fixed operating or entry cost breaks the homogeneity and make the customer base of fringe sellers a state variable. These natural extensions are thus likely to break the above approximate neutrality result.[24] A thorough treatment of entry and exit, however, is beyond the scope of the paper and left for future work.

4.2 Introduction of Granular Seller

This section computes the transition dynamics of an economy after the introduction of a granular seller. Figure 6 plots the corresponding impulse responses. Impulse responses of prices are in percentage deviations from initial values while the impulse responses of the remaining variables are in terms of total deviations from initial values. Initially, fringe sellers are at their stationary equilibrium. A granular seller is introduced with a customer base of $m_l = 0.01$. Convergence to the stationary equilibrium with a large seller is slow, taking roughly 250 periods.

The left panel shows the transition dynamics for the variables of the granular seller. At an initially small customer base the granular seller charges low prices to attract customers and over time harvests their granularity to charge higher markups, at first charging markups slightly above one ($\mu_{l,0} = 1.24$) to increase them along the transition by nearly 40% ($\mu_{l,T} = 1.73$, $T = 250$). We note that convergence is initially slow

[24] An interesting question that arises in a model with entry and exit or another form of deviating from the homogeneity benchmark is whether the presence of a granular seller is qualitatively different from a model with more fringe sellers. As the arguments above indicate the main channel is through shrinking the customer base of fringe sellers. However, this could be achieved by increasing the measure of fringe sellers. So a modified neutrality result might hold in an extended model.

as granularity is negligible, accelerates as the prominence of the large seller grows with its customer base m_l, and slows down again as the large seller raises its markup.

The middle panel plots the transition dynamics of the markups of fringe sellers with low, medium and high productivity. Responses are overall muted. For the most productive seller prices increase by 1.5% while for the least productive they rise by up to 1% until they converge back to the stationary equilibrium. It might seem counter-intuitive that the entry of a low-price firm raises prices of incumbent fringe firms. However, as the granular seller keeps prices low initially to expand its customer base fringe sellers find it more difficult to attract customers, which reduces their continuation value. By Proposition 5 this lowers the customer investment motive for fringe sellers, which induces them to raise their markups. This effect becomes more pronounced as the customer base of the large seller grows and the granular seller makes up a larger share of fringe sellers' competitors (see Appendix D.1). As a result, the rise in optimal markups is gradual. This effect is counteracted by the granular seller increasing its markup, which makes it easier for fringe sellers to attract customers. This raises their continuation value and induces them to lower prices. In the long-run, forces balance out and we transition to the stationary equilibrium.

The right panel plots the evolution of the distribution λ over time. Blue denotes early periods and red later periods. The most defining feature is that the distribution is virtually constant along the transition, which is suggestive of the partially oblivious equilibrium in fact being a good approximation to the full solution (also see Appendix C.3).

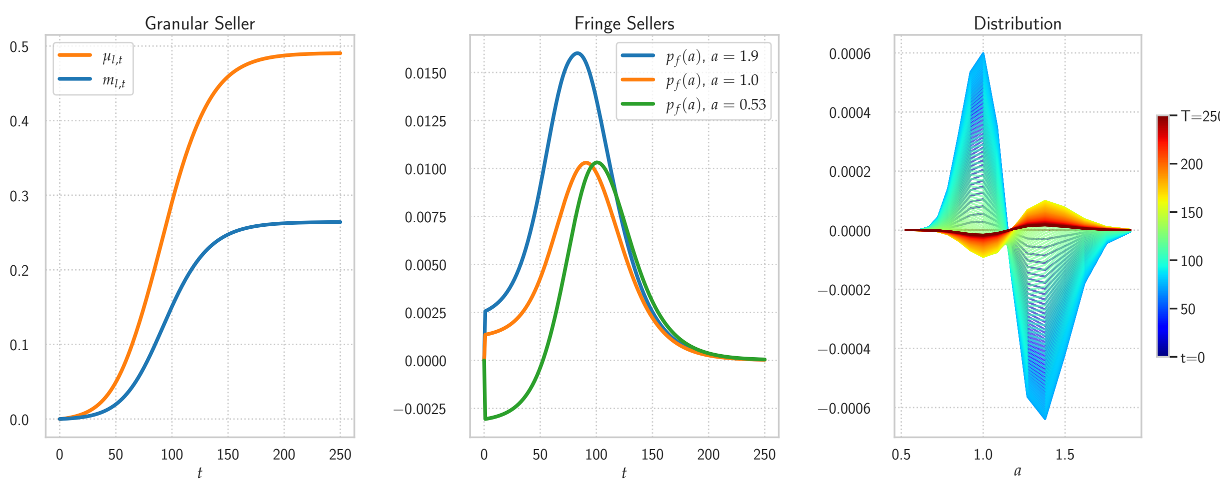

Figure 6: Transition Dynamics of Introduction of a Granular Seller.
Left: absolute deviations from initial values. *Middle:* percentage deviations from initial prices. *Right:* absolute deviations from initial distribution.

4.3 Alternative Matching Process

In the previous sections, we considered a natural benchmark in which the granularity of the large seller was fully determined by the endogenous variable m_l. In this section we consider another natural benchmark in which the degree of granularity is given exogenously. I modify the model akin to Menzio & Trachter (2015). I replace the cumulative distribution function (2.2) characterising the process of matching with a new firm indexed by i_r with the following expression

$$G(i_r) = \begin{cases} 0 & \text{if } i_r < 0, \\ \frac{1-\alpha}{1-m_l} \int_0^{i_r} m(j)dj & \text{if } 0 \leqslant i_r \leqslant J, \\ 1-\alpha & \text{if } J < i_r < l, \\ 1 & \text{if } i_r \geqslant l, \end{cases} \tag{4.3}$$

where without loss of generality $l > J$. A searching customer matches at the rate $\frac{1-\alpha}{1-m_l} m(j)$ with a fringe seller j and with the granular seller with probability α. The higher the constant parameter α the greater the contact probability of the large seller and the lower the contact rates for fringe sellers. This can be viewed as a greater matching advantage for the granular firm. While full model details are delegated to Appendix A.4 it is instructive to examine the Law of Motion for the customer base of the granular seller to gain some intuition

$$\mathcal{M}_l(p, \hat{p}_f) = m_l + \underbrace{\alpha(1-m_l)\theta \mathbb{E}_\lambda \frac{1}{1+(p/\hat{p}_f)^\sigma}}_{\text{Inflows}} - \underbrace{m_l \theta (1-\alpha) \mathbb{E}_\lambda \frac{(p/\hat{p}_f)^\sigma}{1+(p/\hat{p}_f)^\sigma}}_{\text{Outflows}}. \tag{4.4}$$

The most notable difference to the earlier model is that the matching advantage is no longer endogenous in the customer base. This suggests a different form of search friction. For example, α could stand in as a reduced form parameter for e.g. prominence, network effects, and advertisement spending. As an implication inflows and outflows no longer scale by the same factor. Instead, as $m_l \to 0$ the granular seller experiences no outflows while it maintains the same matching advantage α. This increases incentives to attract customers and lower prices when the customer base is small. At the same time, as $m_l \to 1$ the granular seller cannot attract any customers and thus will surely experience outflows for $\alpha < 1$. By implication, the large seller no longer has monopoly power when $m_l \to 1$. In addition, if $\alpha > m_l$ the large seller experiences a net inflow into their customer base even for $p_l = p_f$. This is another advantage of the large seller in this alternative formulation.

4.3.1 Non-Neutrality of Granular Seller

To illustrate the workings of the model, I consider numerical examples parametrised as in section 3.6 in order to ensure comparability. The model output for three different $\alpha \in \{0.1, 0.3, 0.5\}$ is reported in Table 2. The fixed matching advantage is a powerful source of market power. Markups of a granular seller are significantly higher than in the case of the fully endogenous matching process, yet roughly in line with, though slightly higher than, the 90th percentile in the markup distribution as documented by De Loecker et al. (2020). Notably, the rise in markups does not translate into a significant loss in terms of the large seller's customer base.[25]

Further, note that the average markup of fringe sellers varies by α, indicating that the steady state behaviour of fringe sellers is not neutral to the presence of a granular seller. In addition, the Kullback-Leibler divergence measure $D_{KL}(\pi||\lambda)$ is lower for higher α, which implies a lower reallocation of customers from low to high productivity firms for higher α. This is also reflected in higher doubling times (DT-90) and half-lives (HT-10) of firms in the 90th and 10th percentile of the growth distribution, respectively, the higher α – indicating a decline in growth rates at the top due to the presence of the large seller.

	μ_l	$\mathbb{E}_\lambda[\mu_f]$	$\mathbb{E}[\mu_i]$	$m_{l,ss}$	$\frac{p_l}{(\mathbb{E}_\lambda p_f^\sigma)^{1/\sigma}}$	$\mathbb{V}ar_\lambda(p_f^\sigma)$	$D_{KL}(\pi\|\|\lambda)$	DT-90	HL-10
Baseline	1.73	1.55	1.59	0.27	0.97	0.13	0.61	13.94	8.66
$\alpha = 0.1$	2.51	1.55	1.62	0.07	1.39	0.13	0.57	14.03	9.06
$\alpha = 0.3$	2.86	1.58	1.85	0.21	1.55	0.12	0.49	14.39	10.34
$\alpha = 0.5$	3.38	1.62	2.25	0.36	1.76	0.12	0.37	15.27	12.79

Table 2: Summary Statistics of Stationary Equilibrium under Alternative Matching Process.

Note: Baseline: corresponds to model with endogenous matching advantage from section 3.6. DT-90: Doubling time of firm in the 90th percentile of the growth distribution. HL-10: Half-life of firm in the 10th percentile of the growth distribution.

Figure 7 compares the stationary equilibria of an economy with and an economy without a granular seller and graphically illustrates the results from Table 2. The neutrality is broken by two effects. First, fringe sellers tend to charge higher markups as the presence of a large seller makes it harder for them to be seen and attract new customers. Second, since the granular seller charges higher prices unproductive firms lower markups (strategic substitutes) while productive fringe firms raise markups (strategic complements). Further details are presented in Appendix E.

Two consequences follow. Relative to an economy without a granular seller customers are reallocated from high productivity fringe firms to low productivity ones. Further, high productivity firms experience relatively lower growth while low productivity firms are slower to exit. Hence, the presence of a granular seller lowers firm dynamism under this alternative matching process and inhibits the reallocation of customers towards high productivity firms.

To understand why the stationary equilibrium under the alternative matching process differs from the baseline equilibrium consider the case when $m_l = \alpha$ initially. Note that in the alternative model there is no

[25] To see why consider the case in which $p_l = p_f$ and $\sigma_a = 0$. Then by (4.4) $\mathcal{M}_l(p_l, \hat{p}_f) = m_l + (1/2)(\alpha - m_l)$. Hence, for $m_l < \alpha$ the granular seller is able to charge higher prices while maintaining net inflows into their customer base.

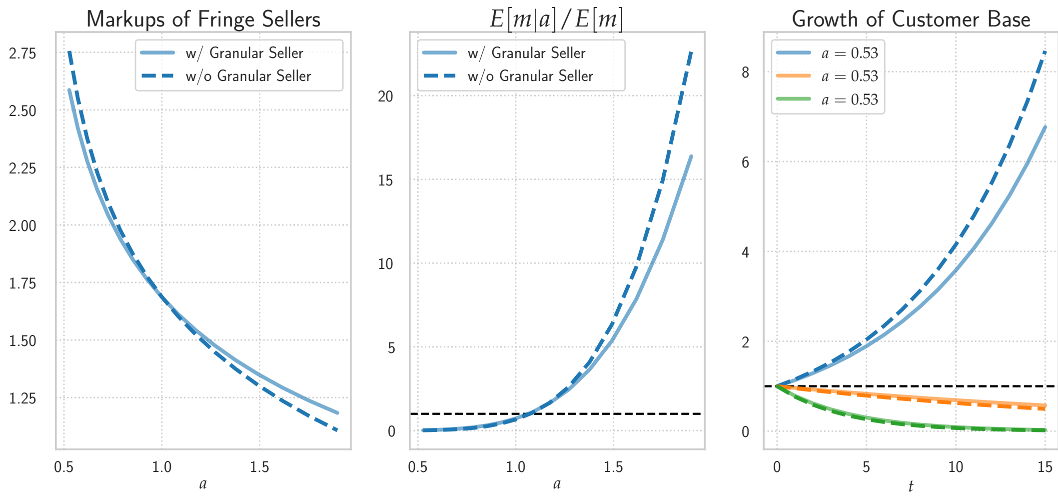

Figure 7: Non-Neutrality under Alternative Matching Process.

trade-off between raising prices and maintaining a large matching advantage as the latter is fixed. Absent this trade-off the large seller raises their prices by more than in the baseline model, causing outflows out of the customer base and leading to an equilibrium in which $m_{l,ss} < \alpha$.[26]

Intuition. To understand why the alternative matching process breaks the neutrality result even for small price dispersion, it is instructive to consider the limiting case where all fringe sellers are of one type. Recall that in this case the model with fully endogenous matching advantage features exact neutrality under the presence of a granular seller. The following Lemma characterises the equilibrium.

Lemma 4. *Suppose the alternative matching technology as defined in (4.3) and that there is only one type of fringe sellers A_f and one type of granular seller A_l.*

1. *The steady state condition for the customer base of the large seller implies*

$$p_l(m_{l,ss}) = \left[\frac{\alpha}{1-\alpha}\frac{1-m_{l,ss}}{m_{l,ss}}\right]^{1/\sigma} p_f(m_{l,ss}). \tag{4.5}$$

2. *Exact neutrality under the presence of a granular seller requires $p_l(m_{l,ss}) = p_f(m_{l,ss})$. This is an equilibrium outcome if and only if the following knife-edge condition holds*

$$\frac{A_l - A_f}{A_f} = \frac{(1-\beta)\alpha + \frac{1}{2}\beta\theta}{\sigma\theta/2 + (1-\beta)}\frac{1}{1-\alpha}. \tag{4.6}$$

Proof. See Appendix B.10. □

[26]This intuition holds for plausible calibrations but may break down for a very large productivity lead of the granular seller.

Lemma 4 highlights two differences to the model with fully endogenous matching advantage. First, (4.5) shows that for exact neutrality to hold, i.e. $p_l(m_{l,ss}) = p_f(m_{l,ss})$, the steady state customer base of the granular seller must take a specific value $m_{l,ss} = \alpha$. In contrast, in the model with fully endogenous matching advantage the steady state condition did not depend directly on the customer base $m_{l,ss}$. Any $m_{l,ss}$ that ensures $p_l(m_{l,ss}) = p_f(m_{l,ss})$ was admissible. Further, the second point of the Lemma illustrates that exact neutrality is only achieved under a particular productivity lead of the large seller. In that case, incentives for the granular seller balance in just the right way such that endogenously $m_{l,ss} = \alpha$. However, away from this knife-edge case neutrality breaks down.[27] Hence, in general the presence of a granular seller under the alternative matching process distorts the stationary pricing behaviour of fringe sellers as well as their expected growth. Numerical simulations confirm that the insights from the Lemma extend to the case of heterogeneous productivity.

5 Conclusion

In this paper, I study the implications of the presence of a dominant granular firm for the aggregate economy and the growth of smaller fringe firms. I develop a model with product market search in which granular firms have an advantage in matching with customers due to their greater *visibility* and in which fringe firms are harder to find by shoppers due to their infinitesimal size. As a natural implication of the matching process large sellers are endowed with additional market power thanks to their granularity which they use to charge high markups. Thus, the emergence of granular firms may have been an important contributor to the observed rise in aggregate markups and in particular at the top of the distribution.

The dynamic implications for firm dynamism of fringe firms are less clear. In a natural benchmark model with endogenous matching advantage I derive an approximate neutrality result which states that the behaviour of fringe sellers is identical in a model with versus a model without a large seller. I then extend this benchmark and show that an alternative matching process carries the novel implications that the gravitational pull of a granular seller diverts matches away from high-productivity fringe sellers, making them harder to find, and reducing their growth potential.

This paper can be extended in many directions, both in terms of theory and empirics. First, the model in this paper can be embedded in a multi-sector economy with endogenous entry and exit and carefully calibrated to the data. Second, the fixed matching advantage from section 4.3 can be endogenised by, for example, explicitly modelling the advertising decision of firms. Third, the matching process can be extended to one of directed search along the lines of Sun (2021). Fourth, the role of aggregate risk stemming from fluctuations in the large firm's productivity may be another interesting pursuit. On the empirical side, it is left to test the theoretical implications presented in this paper empirically. Exploiting the cross-

[27] We note that if $A_l >> A_f$, i.e. greater than the threshold, the granular seller induces fringe sellers to lower prices, but itself charges higher markups.

sectional variation between product markets in terms of search frictions and top concentration seems a natural starting point.

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A Model Derivations and Additional Results

A.1 Law of Motion of Customer Base with Heterogeneous Fringe Firms

Granular Seller. Recall from Lemma 1 and its proof in Appendix (B.1) that

$$\begin{aligned}
\mathcal{M}_l(p_l, p_f) &= m_l - m_l\theta \int_0^J m(j) \frac{(p_l/p_f(j))^\sigma}{1+(p_l/p_f(j))^\sigma} dj + \int_0^J m(j)\theta m_l \frac{1}{1+(p_l/p_f(j))^\sigma} dj \\
&= m_l \left(1 + \theta \int\int m \frac{1-(p_l/p_f(a))^\sigma}{1+(p_l/p_f(a))^\sigma} d\Lambda(a,m)\right) \\
&= m_l \left(1 + (1-m_l)\theta \int \frac{1-(p_l/p_f(a))^\sigma}{1+(p_l/p_f(a))^\sigma} \left[\frac{1}{1-m_l}\int m\Lambda(a,m)dm\right] da\right) \\
&= m_l \left(1 + (1-m_l)\theta \int \frac{1-(p_l/p_f(a))^\sigma}{1+(p_l/p_f(a))^\sigma} \lambda(a) da\right) \qquad \text{(A.1)}
\end{aligned}$$

where we have used the fact that the policy functions of fringe sellers are independent of their customer base.

Fringe Sellers. Likewise, for a fringe seller $j \in [0, J]$

$$\begin{aligned}
\Delta_j^f(p(j), p_f, p_l) &= 1 - m_l\theta \frac{1-(p(j)/p_l)^\sigma}{1+(p(j)/p_l)^\sigma} + \theta \int_0^J m(j') \frac{1-(p(j)/p_f(j'))^\sigma}{1+(p(j)/p_f(j'))^\sigma} dj' \\
&= 1 - m_l\theta \frac{1-(p(j)/p_l)^\sigma}{1+(p(j)/p_l)^\sigma} + \theta \int\int m \frac{1-(p(j)/p_f(a))^\sigma}{1+(p(j)/p_f(a))^\sigma} d\Lambda(a,m) \\
&= 1 - m_l\theta \frac{1-(p(j)/p_l)^\sigma}{1+(p(j)/p_l)^\sigma} + (1-m_l)\theta \int \frac{1-(p(j)/p_f(a))^\sigma}{1+(p(j)/p_f(a))^\sigma} \lambda(a) da \qquad \text{(A.2)}
\end{aligned}$$

A.2 Value Function of Fringe Sellers

For any conjecture of the policy function of the granular seller $\hat{p}_l(m_l)$ and the fringe sellers $\hat{p}(m, a, m_l)$ the full problem of fringe seller j is

$$W_j(m(j), a, m_l) = \max_p m(j)\Delta_j^f(p, \hat{p}_f, \hat{p}_l)(p - \frac{w}{a})\frac{I}{p} + \beta \int W_j(m(j)', a', m_l')P(da'|a) \qquad \text{(A.3)}$$

$$\text{s.t. } \Delta_j^f(p, \hat{p}_f, \hat{p}_l) = 1 + \theta m_l \frac{1-(p/\hat{p}_l(m_l))^\sigma}{1+(p/\hat{p}_l(m_l))^\sigma} + \theta \int\int m \frac{1-(p/\hat{p}_f(m,\tilde{a},m_l))^\sigma}{1+(p/\hat{p}_f(m,\tilde{a},m_l))^\sigma} d\Lambda(\tilde{a}, m)$$

$$m(j)' = m(j)\Delta_j^f(p, \hat{p}_f, \hat{p}_l)$$

Let us guess that the value function is linear in the initial customer base, $W_j(m(j), a, m_l) = m(j)V_j(a, m_l)$.

Then,

$$m(j)V_j(a,m_l) = \max_p m(j)\Delta_j^f(p,\hat{p}_f,\hat{p}_l)(p-\frac{w}{a})\frac{I}{p} + \left[m(j)\Delta_j^f(p,\hat{p}_f,\hat{p}_l)\right]\beta\int V_j(a',m_l')P(da'|a) \tag{A.4}$$
$$\text{s.t. } \Delta_j^f(p,\hat{p}_f,\hat{p}_l) = 1+\theta m_l\frac{1-(p/\hat{p}_l(m_l))^\sigma}{1+(p/\hat{p}_l(m_l))^\sigma} + \theta\int\int m\frac{1-(p/\hat{p}_f(m,\tilde{a},m_l))^\sigma}{1+(p/\hat{p}_f(m,\tilde{a},m_l))^\sigma}d\Lambda(\tilde{a},m)$$

Our guess is verified as the customer base only enters the Bellman equation linearly. Hence, policy functions of fringe sellers are independent of their customer base.

A.3 Stationary Distribution

Granular Seller: From the law of motion of the customer base of the granular seller it is obvious that

$$m_{l,ss} = m_{l,ss}\left(1+(1-m_{l,ss})\theta\int_0^\infty \frac{1-(p_l(m_{l,ss})/p_f(a,m_{l,ss}))^\sigma}{1+(p_l(m_{l,ss})/p_f(a,m_{l,ss}))^\sigma}\lambda(a)da\right) \tag{A.5}$$

implies the condition that for any $m_{l,ss}\in(0,1)$ it must hold that

$$\int_0^\infty \frac{1-(p_l(m_{l,ss})/p_f(a,m_{l,ss}))^\sigma}{1+(p_l(m_{l,ss})/p_f(a,m_{l,ss}))^\sigma}\lambda(a)da = 0 \tag{A.6}$$

Fringe Sellers: Recall that the probability measure $\lambda(a)$ denotes the measure of customers that are attached to fringe sellers of productivity level a. To compute the law of motion for the probability measure we need to keep track of all possible transitions. First, since policy functions are independent of the customer base size each fringe seller with the same productivity charges the same prices and hence experiences the same evolution of their customer base. Thus, the total measure of customers attached to a fringe seller of proctivity a at the end of the period is given by $(1-m_l)\lambda(a)\Delta_j^f(p_f(a,m_l),p_f,p_l)$. Transitioning to the next period the end of period measures are redistributed according to the exogenous stochastic process for a, $P(a'|a)$. Hence, for all measurable Borel sets $\mathscr{A}\subset\mathbb{R}^+$

$$(1-m_{l,ss})\lambda(\mathscr{A}) = \int(1-m_{l,ss})\lambda(\tilde{a})\Delta_j^f(p_f(\tilde{a},m_{l,ss}),p_f,p_l)P(\mathscr{A}|\tilde{a})d\tilde{a} \tag{A.7}$$

which after cancelling terms

$$\lambda(\mathscr{A}) = \int\lambda(\tilde{a})\Delta_j^f(p_f(\tilde{a},m_{l,ss}),p_f,p_l)P(\mathscr{A}|\tilde{a})d\tilde{a} \tag{A.8}$$

Alternative Derivation. The Law of Motion for the joint density is

$$\Lambda'(m,a) = \int\int\mathbf{1}(\tilde{m}\Delta_j^f(\tilde{a}) = m)\Lambda(\tilde{m},\tilde{a})P(a|\tilde{a})d\tilde{a}d\tilde{m} = \int P(a|\tilde{a})\Lambda\big(m/\Delta_j^f(\tilde{a}),\tilde{a}\big)d\tilde{a} \tag{A.9}$$

Then by definition

$$\begin{aligned}\lambda'(a) &= \frac{1}{1-m_l'} \int m\Lambda'(a,m)dm \\ &= \frac{1}{1-m_l'} \int\int P(a|\tilde{a})\Lambda\big(m/\Delta_j^f(\tilde{a}),\tilde{a}\big)d\tilde{a}dm \\ &= \frac{1}{1-m_l'} \int \Delta_j^f(\tilde{a})P(a|\tilde{a}) \left[\int \tilde{m}\Lambda(\tilde{m},\tilde{a})d\tilde{m}\right] d\tilde{a} \\ &= \frac{1-m_l}{1-m_l'} \int \Delta_j^f(\tilde{a})P(a|\tilde{a})\lambda(\tilde{a})d\tilde{a} \end{aligned} \tag{A.10}$$

where the third line uses a change of variable $m = \tilde{m}\Delta_j^f$ and the fourth line uses the definition for λ. In the steady state $m_l = m_l'$ and the expression reduces to the one above.

A.4 Alternative Matching Advantage

For the large seller

$$\begin{aligned}\mathcal{M}_l(p_l,p_f) &= m_l + \alpha\int_0^J m(j')\theta\frac{1}{1+(p_l/p_f(j'))^\sigma}dj' - m_l\theta\int_0^J \frac{1-\alpha}{1-m_l}m(j')\frac{(p_l/p_f(j'))^\sigma}{1+(p_l/p_f(j'))^\sigma}dj' \\ &= m_l + \alpha(1-m_l)\theta\int_0^\infty \frac{1}{1+(p_l/p_f(a))^\sigma}\lambda(a)da - m_l\theta(1-\alpha)\int_0^\infty \frac{(p_l/p_f(a))^\sigma}{1+(p_l/p_f(a))^\sigma}\lambda(a)da \end{aligned} \tag{A.11}$$

which uses the fact that the contact rate with a fringe seller j' is $\frac{1-\alpha}{1-m_l}m(j')$.

Likewise for the fringe sellers

$$\begin{aligned}\mathcal{M}_j^f(p_f(j),p_f,p_l) &= m(j) + m_l\theta\frac{1-\alpha}{1-m_l}m(j)\frac{(p_l/p_f(j))^\sigma}{1+(p_l/p_f(j))^\sigma} \\ &\quad + \int_0^J m(j')\theta\frac{1-\alpha}{1-m_l}m(j)\frac{1}{1+(p_f(j)/p_f(j'))^\sigma}dj' \\ &\quad - m(j)\theta\alpha\frac{1}{1+(p_l/p_f(j))^\sigma} \\ &\quad - m(j)\theta\int_0^J \frac{1-\alpha}{1-m_l}m(j')\theta\frac{(p_f(j)/p_f(j'))^\sigma}{1+(p_f(j)/p_f(j'))^\sigma}dj \\ &= m(j)\Bigg\{1 + m_l\theta\frac{1-\alpha}{1-m_l}\frac{(p_l/p_f(j))^\sigma}{1+(p_l/p_f(j))^\sigma} - \theta\alpha\frac{1}{1+(p_l/p_f(j))^\sigma} \\ &\quad + (1-\alpha)\theta\int_0^\infty \frac{1-(p_f(j)/p_f(a))^\sigma}{1+(p_f(j)/p_f(a))^\sigma}\lambda(a)da\Bigg\} \qquad \text{(A.12)} \\ &= m(j)\Delta_j^f(p_f(j),p_f,p_l) \qquad \text{(A.13)}\end{aligned}$$

Given these expression we can plug them into (3.4) and (3.5) and solve the problem numerically, mutatis mutandis. In particular, this holds as it is easily verified that the state space of fringe sellers can be reduced to abstract from changes to $m(j)$.

B Omitted Proofs

B.1 Proof of Lemma 1

Proof. **Outflows.** With an abuse of notation let $Pr[i]$ denote the probability of meeting a seller with index i and $Pr[i \mapsto i'|i']$ the probability of a customer switching to seller i' from its initial attachment i conditional on having matched with i'. The notation refers both to probability masses and densities. Recall that customers initially attached to the large seller but searching for an alternative meet with fringe seller j at the rate $m(j)$. As they always have the option to return to their initial match outflows are

$$\begin{aligned}
\mathcal{O}_l &= \int_0^J Pr[l \mapsto j|j] Pr[j] dj \\
&= \int_0^J Pr[U_j^h > U_j^h] m(j) dj \\
&= \int_0^J Pr\left[\epsilon_l^h - \epsilon_j^h < \sigma \log(p_l/p_f(j))\right] m(j) dj \\
&= \int_0^J m(j) \frac{e^{\sigma \log(p_l/p_f(j))}}{1 + e^{\sigma \log(p_l/p_f(j))}} dj \\
&= \int_0^J m(j) \frac{(p_l/p_f(j))^\sigma}{1 + (p_l/p_f(j))^\sigma} dj
\end{aligned} \tag{B.1}$$

where the first line uses the definition of outflows that the fraction of individuals leaving, conditional on not having *re*-matched with the granular seller, is the integral over all the possibilities of matching with a fringe seller times the fraction that experience sufficiently low utility from the granular seller to switch. The second and third lines plug in the definitions. The fourth line uses the assumption that taste shocks are distributed according to the Gumbel extreme value distribution. The fifth line simplifies.

Inflows. Conditional on a searching customer initially matched with fringe seller j the fraction of customers switching to the large seller is

$$\begin{aligned}
\mathcal{I}_l(j) &= Pr[j \mapsto l|l] Pr[l] \\
&= Pr[U_l^h > U_j^h] m_l \\
&= \int_0^J Pr\left[\epsilon_j^h - \epsilon_l^h < -\sigma \log(p_l/p_f(j))\right] m_l \\
&= m_l \frac{e^{-\sigma \log(p_l/p_f(j))}}{1 + e^{-\sigma \log(p_l/p_f(j))}} \\
&= m_l \frac{1}{1 + (p_l/p_f(j))^\sigma}
\end{aligned} \tag{B.2}$$

Suppose a symmetric equilibrium amongst fringe sellers. Then $p_f(j) = p_f$ for all $j \in [0, J]$. The expression for the end of period customer base for the large seller reduces to

$$\mathcal{M}_l(p_l, p_f) = m_l - m_l\theta \int_0^J m(j) \frac{(p_l/p_f)^\sigma}{1+(p_l/p_f(j))^\sigma} dj + \int_0^J m(j)\theta m_l \frac{1}{1+(p_l/p_f)^\sigma} dj$$
$$= m_l - m_l\theta(1-m_l)\frac{(p_l/p_f)^\sigma}{1+(p_l/p_f)^\sigma} + (1-m_l)\theta m_l \frac{1}{1+(p_l/p_f)^\sigma} \tag{B.3}$$

since $\int_0^J m(j)dj = 1 - m_l$. □

B.2 Proof of Lemma 2

Proof. The derivations are a straightforward extension from Lemma 1.

Outflows.

$$\begin{aligned} \mathcal{O}_f(j) &= Pr[j \mapsto \neg j] \\ &= Pr[j \mapsto l|l]Pr[l] + \int_0^J Pr[j \mapsto l|j']Pr[j']dj' \\ &= Pr[U_j^h < U_l^h]m_l + \int_0^J Pr[U_j^h < U_{j'}^h]m(j')dj' \\ &= m_l \frac{(p_f(j)/p_l)^\sigma}{1+(p_f(j)/p_l)^\sigma} + \int_0^J m(j')\frac{(p_f(j)/p_f(j'))^\sigma}{1+(p_f(j)/p_f(j'))^\sigma} dj' \end{aligned} \tag{B.4}$$

Inflows.

$$\begin{aligned} \mathcal{I}_{ff}(j.j') &= Pr[j' \mapsto j] \\ &= Pr[j' \mapsto j|j]Pr[j] \\ &= m(j)\frac{1}{1+(p_f(j)/p_f(j'))^\sigma} \end{aligned} \tag{B.5}$$

and

$$\begin{aligned} \mathcal{I}_{lf}(j) &= Pr[l \mapsto j] \\ &= Pr[l \mapsto j|j]Pr[j] \\ &= m(j)\frac{1}{1+(p_f(j)/p_l)^\sigma} \end{aligned} \tag{B.6}$$

Supposing a symmetric equilibrium from all fringe sellers. Then $p_f(j') = p_f$ for all $j' \in [0, J]$ and $j \neq j'$.

Hence,

$$\Delta_j^f(p(j), p_f, p_l) = 1 - \theta\left[m_l \frac{(p_f(j)/p_l)^\sigma}{1+(p_f(j)/p_l)^\sigma} + \int_0^J m(j')\frac{(p_f(j)/p_f)^\sigma}{1+(p_f(j)/p_f)^\sigma}dj'\right]$$
$$+ \int_0^J m(j')\theta\frac{1}{1+(p_f(j)/p_f)^\sigma}dj' + m_l\theta\frac{1}{1+(p_f(j)/p_l)^\sigma}$$
$$= 1 - \theta m_l \frac{(p_f(j)/p_l)^\sigma}{1+(p_f(j)/p_l)^\sigma} + m_l\theta\frac{1}{1+(p_f(j)/p_l)^\sigma}$$
$$- \theta(1-m_l)\frac{(p_f(j)/p_f)^\sigma}{1+(p_f(j)/p_f)^\sigma} + (1-m_l)\frac{1}{1+(p_f(j)/p_f)^\sigma} \tag{B.7}$$

□

B.3 Proof of Proposition 1

Proof. In the static model the optimal markup takes the standard form

$$\frac{p_l}{w/A_l} = \frac{\varepsilon_{\mathcal{M}_l}+1}{\varepsilon_{\mathcal{M}_l}} \tag{B.8}$$

where $\varepsilon_{\mathcal{M}_l} = -\frac{d\log\mathcal{M}_l(p,\hat{p}_f)}{d\log p}$. We can rewrite this as

$$\frac{p_l}{w/A_l} = 1 + \frac{\mathcal{M}_l(p_l,\hat{p}_f)}{-\frac{\partial\mathcal{M}_l(p,\hat{p}_f)}{\partial p}p_l} \tag{B.9}$$

We know that

$$-\frac{\partial\mathcal{M}_l(p,\hat{p}_f)}{\partial p}p_l = m_l 2\sigma\theta(1-m_l)\frac{(p_l/\hat{p}_f)^\sigma}{[1+(p_l/\hat{p}_f)^\sigma]^2} \tag{B.10}$$

To ease the notational burden denote by $x = (p_l/\hat{p}_f)^\sigma$. Then,

$$\frac{p_l}{w/A_l} = 1 + \frac{1+\theta(1-m_l)\frac{1-x}{1+x}}{2\sigma\theta(1-m_l)\frac{x}{[1+x]^2}}$$
$$= 1 + \frac{(1+x)(1+x^{-1}) + \theta(1-m_l)(1-x)(1+x^{-1})}{2\sigma\theta(1-m_l)}$$
$$= 1 + \frac{1 + \frac{1-\theta(1-m_l)}{2}x + \frac{1+\theta(1-m_l)}{2}x^{-1}}{\sigma\theta(1-m_l)} \tag{B.11}$$
$$= 1 + \frac{1}{\sigma\theta(1-m_l)} + \frac{1}{\sigma\theta(1-m_l)}\left(\frac{1-\theta(1-m_l)}{2}x + \frac{1+\theta(1-m_l)}{2}x^{-1}\right)$$
$$= \frac{\sigma+1}{\sigma} + \frac{1}{\sigma}\frac{1-\hat{\theta}}{\hat{\theta}} + \frac{1}{\sigma\hat{\theta}}\left(\frac{1-\hat{\theta}}{2}(p_l/\hat{p}_f)^\sigma + \frac{1+\hat{\theta}}{2}(p_l/\hat{p}_f)^{-\sigma}\right) \tag{B.12}$$

where the last line defines $\hat{\theta} = \theta(1-m_l)$ and plugs back in for $x = (p_l/\hat{p}_f)^\sigma$. □

B.4 Proof of Proposition 2

Proof. As in the proof of Proposition 1 the optimal markup is given by

$$\frac{p_f}{w/A_f} = \frac{\varepsilon_{\mathcal{M}_j^f} + 1}{\varepsilon_{\mathcal{M}_j^f}} \tag{B.13}$$

where $\varepsilon_{\mathcal{M}_j^f} = -\frac{d \log \mathcal{M}_j^f(p, \hat{p}_f, \hat{p}_l)}{d \log p} =$. As before we can compute

$$\begin{aligned}
-\frac{d\mathcal{M}_j^f(p, \hat{p}_f, \hat{p}_l)}{dp} p_f &= 2\sigma\theta m_l m(j) \frac{(\hat{p}_l/p_f)^\sigma}{[1 + (\hat{p}_l/p_f)^\sigma]^2} + 2\sigma\theta(1 - m_l)m(j)\frac{(\hat{p}_f/p_f)^\sigma}{[1 + (\hat{p}_f/p_f)^\sigma]^2} \\
&= m(j)2\sigma\theta \left[m_l \frac{(\hat{p}_l/p_f)^\sigma}{[1 + (\hat{p}_l/p_f)^\sigma]^2} + (1 - m_l)\frac{1}{4} \right]
\end{aligned} \tag{B.14}$$

Again denoting $x = (\hat{p}_l/p_f)^\sigma$ to ease the notational burden we can plug in

$$\begin{aligned}
\frac{p_f}{w/A_f} &= 1 + \frac{1}{\varepsilon_{\mathcal{M}_j^f}} \\
&= 1 + \frac{\mathcal{M}_j^f(p_f, p_f, \hat{p}_l)}{-\frac{d\mathcal{M}_j^f(p, \hat{p}_f, \hat{p}_l)}{dp} p_f} \\
&= 1 + \frac{1 + \theta m_l \frac{x-1}{1+x}}{2\sigma\theta \left[m_l \frac{x}{[1+x]^2} + (1 - m_l)\frac{1}{4} \right]} \\
&= 1 + \frac{(1 + x)(1 + x^{-1}) + \theta m_l (x - 1)(1 + x^{-1})}{2\sigma\theta \left[m_l + (1 - m_l)\frac{1}{4}(1 + x)(1 + x^{-1}) \right]} \\
&= 1 + 2\frac{1 + \frac{1}{2}(1 + \theta m_l)x + \frac{1}{2}(1 - \theta m_l)x^{-1}}{\sigma\theta \left[(1 + m_l) + \frac{1}{2}(1 - m_l)(x + x^{-1}) \right]} && \text{(B.15)} \\
&= \frac{\sigma + 1}{\sigma} + \frac{1}{\sigma}\frac{1 - \theta}{\theta} + \frac{1}{\sigma\theta}\frac{2 + (1 + \theta m_l)x + (1 - \theta m_l)x^{-1} - (1 + m_l) - \frac{1}{2}(1 - m_l)(x + x^{-1})}{(1 + m_l) + \frac{1}{2}(1 - m_l)(x + x^{-1})} \\
&= \frac{\sigma + 1}{\sigma} + \frac{1}{\sigma}\frac{1 - \theta}{\theta} + \frac{1}{\sigma\theta}\frac{\frac{1-m_l}{1+m_l} + \phi(\hat{p}_l/p_f)^\sigma + (1 - \phi)(\hat{p}_l/p_f)^{-\sigma}}{1 + \frac{1-m_l}{1+m_l}\frac{1}{2}((\hat{p}_l/p_f)^\sigma + (\hat{p}_l/p_f)^{-\sigma})} && \text{(B.16)}
\end{aligned}$$

where $\phi = \frac{1}{2} + \frac{\theta m_l}{1+m_l}$ and we have plugged back in for $x = (\hat{p}_l/p_f)^\sigma$ to get the desired expression. □

B.5 Proof of Proposition 3

Proof. To ease the notational burden in the proof we denote the relative price by $z \equiv \frac{p_l}{p_f}$. From (B.11) and (B.15) we recall that the optimal pricing strategies can be written as

$$\frac{p_l}{w/A_l} = 1 + \frac{1 + \omega z^{-\sigma} + (1-\omega) z^{\sigma}}{\sigma \hat{\theta}} \tag{B.17}$$

$$\frac{p_f}{w/A_f} = 1 + \frac{2}{\sigma \hat{\theta}} \frac{1 + \tilde{\omega} z^{-\sigma} + (1-\tilde{\omega}) z^{\sigma}}{\frac{1+m_l}{1-m_l} + \frac{1}{2}(z^{\sigma} + z^{-\sigma})} \tag{B.18}$$

where $\omega = \frac{1}{2}(1 + (1-m_l)\theta)$, $\tilde{\omega} = \omega - \frac{1}{2}\theta = \frac{1}{2}(1 - m_l\theta)$ and $\hat{\theta} = (1-m_l)\theta$.

Combining the two equilibrium equations we get

$$\left(z^{-1} + \frac{z^{-1} + \omega z^{-\sigma-1} + (1-\omega) z^{\sigma-1}}{\sigma \hat{\theta}} \right) \frac{A_f}{A_l} = 1 + \frac{2}{\sigma \hat{\theta}} \frac{1 + \tilde{\omega} z^{-\sigma} + (1-\tilde{\omega}) z^{\sigma}}{\frac{1+m_l}{1-m_l} + \frac{1}{2}(z^{\sigma} + z^{-\sigma})} \tag{B.19}$$

Under the parameter restriction that $\sigma < 1$ it is straightforward to note that the LHS of (B.19) is decreasing in z. The proof will proceed in 6 steps. In step 1. we prove existence. In step 2. we derive a sufficient condition for a unique equilibrium. Steps 3. and 4. derive the main result underlying Point 1 in the Proposition. Step 5. extends the result and Step 6. considers limiting cases.

Step 1. We will prove that there exists a finite $z > 0$ solving the non-linear system (B.19). First, note that

$$\lim_{z \to \infty} LHS = 0 \tag{B.20}$$

$$\lim_{z \to \infty} RHS = 1 + \lim_{z \to \infty} \frac{2}{\sigma \hat{\theta}} \frac{z^{-\sigma} + \tilde{\omega} z^{-2\sigma} + 1 - \tilde{\omega}}{\frac{1+m_l}{1-m_l} z^{-\sigma} + \frac{1}{2}(1 + z^{-2\sigma})} = 1 + \frac{4(1-\tilde{\omega})}{\sigma \hat{\theta}} > 0 \tag{B.21}$$

and likewise

$$\lim_{z \to 0} LHS = \infty \tag{B.22}$$

$$\lim_{z \to 0} RHS = 1 + \lim_{z \to 0} \frac{2}{\sigma \hat{\theta}} \frac{z^{\sigma} + \tilde{\omega} + (1-\tilde{\omega}) z^{2\sigma}}{\frac{1+m_l}{1-m_l} z^{\sigma} + \frac{1}{2}(z^{2\sigma} + 1)} = 1 + \frac{4\tilde{\omega}}{\sigma \hat{\theta}} < \infty \tag{B.23}$$

By continuity of (B.19) in z an appeal to the Intermediate Function Theorem proves **existence**.

Step 2. Since it is assumed that $\sigma < 1$ we have already noted that the LHS of (B.19) is decreasing in z. Given the previous existence result a sufficient condition for equilibrium uniqueness is for the RHS of (B.19) to be upward sloping. The derivative of the RHS with respect to the relative price z is proportional to

$$\frac{dRHS}{dz} \propto - \left[\frac{1}{2} \left(z^{-\sigma} - z^{\sigma} \right) - \frac{1}{2} m_l \theta \left(z^{-\sigma} + z^{\sigma} \right) \right] \left(\frac{1+m_l}{1-m_l} + \frac{1}{2} \left(z^{-\sigma} + z^{\sigma} \right) \right)$$

$$
\begin{aligned}
&- \frac{1}{2}\left(z^{\sigma} - z^{-\sigma}\right)\left(1 + \tilde{\omega} z^{-\sigma} + (1-\tilde{\omega}) z^{\sigma}\right) \\
&= \frac{m_l}{1-m_l}\left((1+\frac{1}{2}\theta(1+m_l))z^{\sigma} - (1-\frac{1}{2}\theta(1+m_l))z^{-\sigma}\right) + m_l\theta
\end{aligned} \tag{B.24}
$$

We immediately note that for $z > 1$ this derivative is positive. If we could establish $z > 1$ we would directly obtain uniqueness. As $1 - \frac{1}{2}\theta(1+m_l) > 0$ it is straightforward that the expression is monotonically increasing in z for all $z > 0$. It is easy to verify that an appeal to the Intermediate Value Theorem proves that there exists a boundary value $\underline{\Gamma}(m_l, \theta) < 1$ for each $m_l \in (0,1)$ and $\theta \in (0,1)$ such that[28]

$$
(1+\frac{1}{2}\theta(1+m_l))\underline{\Gamma}(m_l,\theta)^{\sigma} - (1-\frac{1}{2}\theta(1+m_l))\underline{\Gamma}(m_l,\theta)^{-\sigma} + (1-m_l)\theta = 0 \tag{B.25}
$$

and for all $z \geqslant \underline{\Gamma}(m_l, \theta)$ the RHS is increasing in z, establishing uniqueness. Solving for the unique positive root yields

$$
\underline{\Gamma}(m_l,\theta) = \left(\frac{\sqrt{1-m_l\theta^2} - \frac{1}{2}\theta(1-m_l)}{1+\frac{1}{2}(1+m_l)\theta}\right)^{1/\sigma} \tag{B.26}
$$

In Step 3. and 4. we will show that in the case where $A_l \geqslant A_f$, then $z > \frac{A_f}{A_l}$. If so, restricting ourselves to $\underline{\Gamma}(m_l, \theta) \leqslant \frac{A_f}{A_l} < z$ is sufficient to ensure equilibrium uniqueness. If $A_l < A_f$ the relative price z is always greater than 1 and equilibrium uniqueness follows immediately and does not require any bound. The latter follows from a straightforward modification of the arguments given in Step 3.

Step 3. Suppose that $A_l > A_f$. We will now show that the granular seller charges a higher markup. Working towards a contradiction suppose that $z < \frac{A_f}{A_l} < 1$ or equivalently $\frac{p_l}{w/A_l} < \frac{p_f}{w/A_f}$. Then,

$$
\begin{aligned}
\left(z^{-1} + \frac{z^{-1} + \omega z^{-\sigma-1} + (1-\omega)z^{\sigma-1}}{\sigma\hat{\theta}}\right)\frac{A_f}{A_l} &> 1 + \frac{1 + \omega z^{-\sigma} + (1-\omega)z^{\sigma}}{\sigma\hat{\theta}} \\
&> 1 + \frac{1 + \tilde{\omega} z^{-\sigma} + (1-\tilde{\omega})z^{\sigma}}{\sigma\hat{\theta}}
\end{aligned}
$$

where the first line applies twice the fact that by assumption $z^{-1}\frac{A_f}{A_l} > 1$. The second line then uses the fact that since $\tilde{\omega} = \omega - \frac{1}{2}\theta < \omega$ and $z^{-\sigma} > z^{\sigma}$ given that $z < \frac{A_f}{A_l} < 1$ we have that $1 + \omega z^{-\sigma} + (1-\omega)z^{\sigma} > 1 + \tilde{\omega} z^{-\sigma} + (1-\tilde{\omega})z^{\sigma}$.

We will next show that $\frac{1+m_l}{1-m_l} + \frac{1}{2}\left(z^{\sigma} + z^{-\sigma}\right) > 2$. Note that since $\sigma < 1$ we have that $z^{\sigma} + z^{-\sigma}$ is a strictly convex function for all $z > 0$ with unique minimum at $z = 1$. Hence,

$$
\frac{1+m_l}{1-m_l} + \frac{1}{2}\left(z^{\sigma} + z^{-\sigma}\right) \geqslant 1 + \frac{1}{2}(1+1) = 2 \tag{B.27}
$$

It thus immediately follows that the LHS of (B.19) exceeds the RHS for all $z = \frac{p_l}{p_f} < \frac{A_f}{A_l} \leqslant 1$. A contradiction

[28] Note that $\underline{\Gamma}(m_l, \theta) < 1$ as we have already previously shown that the equation holds for sure for any $z \geqslant 1$.

to our hypothesis of equilibrium. It follows that if $A_l > A_f$ it must be that $\frac{p_l}{w/A_l} \geqslant \frac{p_f}{w/A_f}$.

Step 4. The proof of the previous step required one of the inequalities to be strict. In step 1 this came from the hypothesis (to be contradicted) that $\frac{p_l}{w/A_l} < \frac{p_f}{w/A_f}$ strictly. If we assume a non-degenerate granular seller with $m_l > 0$ we can get also the strict inequality and can arrive at a contradiction for the hypothesis that $z \leqslant \frac{A_f}{A_l}$ since for $m_l > 0$ it holds that

$$\frac{1+m_l}{1-m_l} + \frac{1}{2}(z^{\sigma} + z^{-\sigma}) > 1 + \frac{1}{2}(z^{\sigma} + z^{-\sigma}) \geqslant 2 \tag{B.28}$$

It follows that for $m_l > 0$ it must be that $\frac{p_l}{w/A_l} > \frac{p_f}{w/A_f}$.

Step 5. To consider the last case suppose that $\frac{A_f}{A_l} > 1$. Working towards a contradiction suppose that $z < \frac{A_l}{A_f}$. Then, as in step 1

$$\left(z^{-1} + \frac{z^{-1} + \omega z^{-\sigma-1} + (1-\omega)z^{\sigma-1}}{\sigma\hat{\theta}}\right)\frac{A_f}{A_l} > 1 + \frac{1 + \omega z^{-\sigma} + (1-\omega)z^{\sigma}}{\sigma\hat{\theta}} \tag{B.29}$$

Continuing with the chain of inequalities and using the fact that $\tilde{\omega} = \omega - \frac{1}{2}\theta$ we arrive at a contradiction if for all $z \geqslant 1$[29]

$$\left(\frac{1+m_l}{1-m_l} + \frac{1}{2}(z^{\sigma} + z^{-\sigma})\right)(1 + \omega z^{-\sigma} + (1-\omega)z^{\sigma}) > 2(1 + \omega z^{-\sigma} + (1-\omega)z^{\sigma}) + \theta(z^{\sigma} - z^{-\sigma}) \tag{B.30}$$

or equivalently

$$\left(\frac{1+m_l}{1-m_l} + \frac{1}{2}(z^{\sigma} + z^{-\sigma}) - 2\right)(1 + \omega z^{-\sigma} + (1-\omega)z^{\sigma}) - \theta(z^{\sigma} - z^{-\sigma}) > 0 \qquad \forall z \geqslant 1 \tag{B.31}$$

Next, we establish that there exists a threshold $\underline{m}$ that solves (B.32) for all $z \geqslant 1$. The LHS is greater than

$$LHS > \left(\frac{3m_l - 1}{1-m_l}\frac{1-\theta(1-m_l)}{2} + \frac{1}{2} - \theta\right) z^{\sigma} \tag{B.32}$$

For any $m_l > 1/\sqrt{3}$ this is positive for all $0 < \theta \leqslant 1$. Hence, the threshold defined in the Proposition exists. The stronger sufficient condition (B.32) can be further solved to provide the upper bound on the threshold $\underline{\theta}$ given in the Proposition. Should the inequality (B.31) hold for any arbitrary m_l we set $\underline{m}(\theta) = 0$.

Step 6. Suppose that $A_l = A_f = A$. Next, we show the limiting cases as $m_l \to 0$ and $m_l \to 1$. Note that $\lim_{m_l \to 0} \omega = \frac{1+\theta}{2}$ and $\lim_{m_l \to 0} \tilde{\omega} = \frac{1}{2}$. Plugging this into (B.19) and taking limits on both sides and denoting $\lim_{m_l \to 0} z = \bar{z}$

$$\bar{z}^{-1} + \frac{\bar{z}^{-1} + \frac{1+\theta}{2}\bar{z}^{-1-\sigma} + \frac{1-\theta}{2}\bar{z}^{\sigma-1}}{\sigma\theta} = 1 + \frac{2}{\sigma\theta} \tag{B.33}$$

[29]Note that we can restrict ourselves to $z \geqslant 1$ as a straightforward extension of Steps 1 and 2.

A simple guess and verify establishes that $\bar{z} = 1$ solves the equation and hence $\lim_{m_l \to 0} \frac{p_l}{p_f} = 1$.

Taking the other limit as $m_l \to 1$ and plugging into (B.19), denoting $\hat{z} = \lim_{m_l \to 1} z$ yields

$$\hat{z}^{-1} + \frac{1}{2}\hat{z}^{-\sigma-1} + \frac{1}{2}\hat{z}^{\sigma-1} = 0 \tag{B.34}$$

which cannot be satisfied by any finite relative price. Hence $\lim_{m_l \to 1} \frac{p_l}{p_f} = \infty$. □

B.6 Proof of Proposition 4

Proof. We will prove each point in turn.

1. From (3.6) it follows directly that if the stochastic process for productivity can only take one value then $p_l(m_{l,ss}) = p_f(m_{l,ss})$. To be more precise we will also need to show that $m_{l,ss} > 0$ and we will do so below.

2. As $p_l(m_{l,ss}) = p_f(m_{l,ss})$
$$\frac{\varepsilon_{\mathcal{M}_{l,ss}} + 1}{\varepsilon_{\mathcal{M}_{l,ss}}} = 1 + \frac{2}{\sigma\theta(1 - m_{l,ss})} \tag{B.35}$$
such that from (3.14)
$$\mu_l = (1-\beta)\left(1 + \frac{2}{\sigma\theta(1 - m_{l,ss})}\right) + \beta \tag{B.36}$$
Likewise,
$$\frac{\varepsilon_{\Delta,ss} + 1}{\varepsilon_{\Delta,ss}} = 1 + \frac{2}{\sigma\theta} \tag{B.37}$$
Further, given that there is only one productivity states the markups will be constant for fringe firms in a stationary equilibrium. Hence, $\Delta_{j,t+k} = 1$ for all $k \geqslant 0$ and by (**??**)
$$\begin{aligned} \mu_f &= (1-\beta)\frac{\varepsilon_{\Delta,ss} + 1}{\varepsilon_{\Delta,ss}} + \beta \\ &= (1-\beta)\left(1 + \frac{2}{\sigma\theta}\right) + \beta \end{aligned} \tag{B.38}$$
Solving for the $m_{l,ss}$ that equates $\mu_l \frac{w}{A_l} = \mu_f \frac{w}{A_f}$ yields the expression in the Proposition.

3. Subtracting (B.38) from (B.36) yields the desired result from the Proposition.
$$\begin{aligned} \mu_l - \mu_f &= (1-\beta)\frac{1}{\sigma}\left[\frac{1}{\theta(1 - m_{l,ss})} - \frac{1}{\theta}\right] \\ &= (1-\beta)\frac{2}{\sigma\theta}\frac{m_{l,ss}}{1 - m_{l,ss}} \\ &= \left(1 + \frac{2(1-\beta)}{\sigma\theta}\right)(A_l/A_f - 1) \end{aligned} \tag{B.39}$$
conditional on $A_l > A_f$ to ensure $m_{l,ss} > 0$.

□

B.7 Proof of Corollary 1

Proof. Taking the first derivative of (3.10) yields the desired result.

$$\frac{dm_{l,ss}}{(1-m_{l,ss})^2} = \left[1 + \frac{\sigma\theta}{2(1-\beta)}\right] d(A_l/A_f) + \frac{A_l - A_f}{A_f}\frac{\sigma}{2(1-\beta)}\left[d\theta + \frac{\theta}{1-\beta}d\beta\right] \tag{B.40}$$

$$d\mu_l = \left[1 + \frac{2(1-\beta)}{\sigma\theta}\right] d(A_l/A_f) - \frac{A_l}{A_f}\frac{2(1-\beta)}{\sigma\theta^2}\left[d\theta + \frac{\theta}{1-\beta}d\beta\right] \tag{B.41}$$

□

B.8 Proof of Proposition 5

Proof. We will prove each point in turn.

1. Recall the problem of the granular seller

$$V_{l,t}(m_{l,t}) = \max_p \mathcal{M}_{l,t}(p, p_{f,t})\left(1 - \frac{w_t/a_{l,t}}{p}\right) I_t + \beta\mathbb{E}_t V_{l,t+1}(\mathcal{M}_l(p, p_{f,t})) \tag{B.42}$$

with FOC

$$\mathcal{M}_{l,t}(p_l, p_{f,t})\mu_{l,t}^{-1} I_t = \left(-\frac{\partial\mathcal{M}_{l,t}(p_l, p_{f,t})}{\partial p_l}p_{l,t}\right)\left[(1-\mu_{l,t}^{-1})I_t + \beta\mathbb{E}_t V'_{l,t+1}(\mathcal{M}_{l,t}(p_{l,t}, p_{f,t}))\right] \tag{B.43}$$

and Envelope condition

$$V_{l,t}(m_{l,t}) = \frac{\partial\mathcal{M}_{l,t}}{\partial m_{l,t}}\left((1-\mu_{l,t}^{-1})I_t + \beta\mathbb{E}_t V'_{l,t+1}(\mathcal{M}_{l,t}(p_{l,t}, p_{f,t}))\right) \tag{B.44}$$

Repeated substitution yields

$$\begin{aligned}\mathcal{M}_{l,t}(p_l, p_{f,t})\mu_{l,t}^{-1} I_t = &\left(-\frac{\partial\mathcal{M}_{l,t}(p_l, p_{f,t})}{\partial p_l}p_{l,t}\right)(1-\mu_{l,t}^{-1})I_t \\ &+ \left(-\frac{\partial\mathcal{M}_{l,t}(p_l, p_{f,t})}{\partial p_l}p_{l,t}\right)\mathbb{E}_t\sum_{\tau=1}^{\infty}\beta^\tau\left[\prod_{k=1}^{\tau}\frac{\partial\mathcal{M}_{l,t+k}}{\partial m_{l,t+k}}\right](1-\mu_{l,t+\tau}^{-1})I_{t+\tau}\end{aligned} \tag{B.45}$$

which can rearranged to read

$$\mu_{l,t} = \frac{1+\varepsilon_{\mathcal{M}_{l,t}}}{\varepsilon_{\mathcal{M}_{l,t}}} - \mathbb{E}_t\sum_{\tau=1}^{\infty}\beta^\tau\left[\prod_{k=1}^{j}\frac{\partial\mathcal{M}_{l,t+k}}{\partial m_{l,t+k}}\right]\frac{\mu_{l,t}}{\mu_{l,t+\tau}}(\mu_{l,t+\tau}-1)\frac{I_{t+\tau}}{I_t} \tag{B.46}$$

where $\varepsilon_{\mathcal{M}_{l,t}} = \frac{\partial\log\mathcal{M}_{l,t}}{\partial\log p_{l,t}}$

2. We note that in a stationary equilibrium

$$\frac{\partial \mathcal{M}_{l,t}}{\partial m_{l,t}} = 1 + (1-2m_{l,ss})\theta \int_0^\infty \frac{1-(p_l(m_{l,ss})/p_f(a,m_{l,ss}))^\sigma}{1+(p_l(m_{l,ss})/p_f(a,m_{l,ss}))^\sigma}\lambda(a)da = 1 \tag{B.47}$$

where the last equality follows from the condition for a stationary equilibrium (3.6). Further, the optimal markup sequence of a large seller will be constant as their state variables are constant and there is no aggregate uncertainty. Hence,

$$\mu_{l,ss} = \frac{1+\varepsilon_{\mathcal{M}_{l,ss}}}{\varepsilon_{\mathcal{M}_{l,ss}}} - \frac{\beta}{1-\beta}(\mu_{l,ss}-1) \tag{B.48}$$

which rearranges to the equation in the Proposition.

3. Recall the problem of a fringe seller j

$$V_{j,t}(a_{j,t}, m_{l,t}) = \max_p \Delta^f_{j,t}(p, p_{f,t}, p_{l,t}) \left[(1 - \frac{w_t/a_{j,t}}{p})I_t + \beta \mathbb{E}\left[V_{j,t+1}(a_{j,t+1}, m_{l,t+1})\right]|a_{j,t}\right] \tag{B.49}$$

with FOC

$$-\frac{\partial \log \Delta_{j,t}(p_{j,t}, p_{f,t}, p_{l,t})}{\partial \log p_{j,t}} \left[(1-\mu_{j,t}^{-1})I_t + \beta \mathbb{E}\left[V_{j,t+1}(a_{j,t+1}, m_{l,t+1})|a_{j,t}\right]\right] = \mu_{j,t}^{-1} I_t \tag{B.50}$$

and Envelope condition

$$\mathbb{E}\left[V_{j,t+1}(a_{j,t+1}, m_{l,t+1})|a_{j,t}\right] = \mathbb{E}\left[\Delta^f_{j,t+1}\left((1-\mu_{j,t+1}^{-1})I_{t+1} + \beta \mathbb{E}\left[V_{j,t+2}|a_{j,t+2}\right]\right)|a_{j,t}\right] \tag{B.51}$$

where arguments have been suppressed for notational convenience. Repeated substitution and re-arranging yields

$$\mu_{j,t} = \frac{\varepsilon_{\Delta,t}+1}{\varepsilon_{\Delta,t}} - \sum_{\tau=1}^{\infty} \beta^\tau \mathbb{E}\left[\left(\prod_{k=1}^{\tau}\Delta_{j,t+k}\right)\frac{\mu_{j,t}}{\mu_{j,t+\tau}}(\mu_{j,t+\tau}-1)\frac{I_{t+\tau}}{I_t}|a_{j,t}\right] \tag{B.52}$$

where $\varepsilon_{\Delta,t} = \frac{\partial \log \Delta^f_{j,t}}{\partial \log p_{j,t}}$

□

B.9 Proof of Lemma 3

Proof. First, note that condition (3.6) for a stationary equilibrium requires

$$\mathbb{E}_\lambda \frac{p_l^\sigma}{p_l^\sigma + p_f^\sigma} = \frac{1}{2} \tag{B.53}$$

A second order approximation around $p_f^\sigma = \mathbb{E}_\lambda p_f^\sigma$ and $p_l^\sigma = \mathbb{E}_\lambda p_f^\sigma$ implies

$$p_l^\sigma - \mathbb{E}_\lambda p_f^\sigma = -\frac{1}{2}\frac{1}{\mathbb{E}_\lambda p_f^\sigma}\mathbb{V}ar_\lambda(p_f^\sigma) + \frac{1}{2}\frac{1}{\mathbb{E}_\lambda p_f^\sigma}(p_l^\sigma - \mathbb{E}_\lambda p_f^\sigma)^2 \tag{B.54}$$

which iterating on the RHS and ignoring terms containing $[\mathbb{V}ar(p_f^\sigma)]^k$ for $k \geqslant 2$ and $(p_l^\sigma - \mathbb{E}_\lambda p_f^\sigma)^k$ for $k \geqslant 3$ approximates to

$$p_l^\sigma = \mathbb{E}_\lambda p_f^\sigma - \frac{1}{2}\frac{1}{\mathbb{E}_\lambda p_f^\sigma}\mathbb{V}ar_\lambda(p_f^\sigma) \tag{B.55}$$

Second, note that

$$\begin{aligned}\Delta_j^f(p, p_f, p_l) - \Delta_{j,-l}^f(p, p_f) &= m_{l,ss}\theta\left(\frac{p_l^\sigma - p^\sigma}{p_l^\sigma + p^\sigma} - \mathbb{E}_\lambda \frac{p_f^\sigma - p^\sigma}{p_f^\sigma + p^\sigma}\right) \\ &= 2m_{l,ss}\theta\left(\mathbb{E}_\lambda \frac{p^\sigma}{p_f^\sigma + p^\sigma} - \frac{p^\sigma}{p_l^\sigma + p^\sigma}\right) \end{aligned} \tag{B.56}$$

Taking a second order approximation around $p_l^\sigma = \mathbb{E}_\lambda p_f^\sigma$ and $p_f^\sigma = \mathbb{E}_\lambda p_f^\sigma$, ignoring higher order terms, yields

$$\begin{aligned}\Delta_j^f - \Delta_{j,-l}^f &= 2m_{l,ss}\theta\left(\frac{p^\sigma}{(\mathbb{E}_\lambda p_f^\sigma + p^\sigma)^3}\mathbb{V}ar_\lambda(p_f^\sigma) + \frac{p^\sigma}{(\mathbb{E}_\lambda p_f^\sigma + p^\sigma)^2}(p_l^\sigma - \mathbb{E}p_f^\sigma)\right) \\ &= m_{l,ss}\theta\frac{p^\sigma(\mathbb{E}_\lambda p_f^\sigma - p^\sigma)}{\mathbb{E}_\lambda p_f^\sigma(\mathbb{E}_\lambda p_f^\sigma + p^\sigma)^3}\mathbb{V}ar_\lambda(p_f^\sigma)\end{aligned} \tag{B.57}$$

and hence

$$|\Delta_j^f - \Delta_{j,-l}^f| \leqslant m_{l,ss}\theta\frac{p^\sigma \max\{\mathbb{E}_\lambda p_f^\sigma, p^\sigma\}}{\mathbb{E}_\lambda p_f^\sigma(\mathbb{E}_\lambda p_f^\sigma + p^\sigma)^3}\mathbb{V}ar_\lambda(p_f^\sigma) \tag{B.58}$$

$$\leqslant m_{l,ss}\theta\frac{\mathbb{V}ar_\lambda(p_f^\sigma)}{[\mathbb{E}_\lambda p_f^\sigma]^2} \tag{B.59}$$

□

B.10 Proof of Lemma 4

Proof. **1.** Recall the equations for extensive demand (A.11). The condition for a stationary equilibrium (3.6) directly implies that the following must hold

$$\mathbb{E}_\lambda\left[\frac{1}{1 + (p_l/p_f)^\sigma}\right] = \frac{(1-\alpha)m_{l,ss}}{(1-\alpha)m_{l,ss} + \alpha(1 - m_{l,ss})} \tag{B.60}$$

which, if the productivity distribution is degenerate, reduces to the expression given in the main text.

2. Suppose there is only one productivity type for fringe sellers. As seen in Proposition 4 exact neutrality holds in the case of a single productivity type if $p_l = p_f$. From (B.60) the condition implies that $m_{l,ss} = \alpha$ and combining it with (A.12) confirms that this is also the condition of exact neutrality in the modified model with a fixed matching advantage. In the proof below we will derive conditions under which neutrality is achieved in the modified model. Recall the equations for extensive demand (A.11) and (A.12). At a steady state

$$\begin{aligned}\frac{\partial \mathcal{M}_l}{\partial m_l} &= 1 - \alpha\theta \frac{1}{1+(p_l/p_f)^\sigma} - \theta(1-\alpha)\frac{(p_l/p_f)^\sigma}{1+(p_l/p_f)^\sigma} \\ &= 1 - \frac{1}{2}\theta \end{aligned} \tag{B.61}$$

where the second line uses the condition for exact neutrality $p_l = p_f$. For the fringe seller

$$\Delta_j^f = 1 \tag{B.62}$$

From Proposition 5, mutatis mutandis,

$$\mu_l = (1-\tilde{\beta})\frac{1+\varepsilon_{\mathcal{M}_l}}{\varepsilon_{\mathcal{M}_l}} + \tilde{\beta} \tag{B.63}$$

$$\mu_f = (1-\beta)\frac{1+\varepsilon_{\Delta_j^f}}{\varepsilon_{\Delta_j^f}} + \beta \tag{B.64}$$

where $\tilde{\beta} = \beta(1-\frac{1}{2}\theta) < \beta$, implying that the granular seller places a larger weight on static markups and less weight on the perfect competition markup.[30] We can then solve

$$\begin{aligned}1+\frac{1}{\varepsilon_{\mathcal{M}_l}} &= 1+\frac{1}{\sigma\theta}\frac{m_{l,ss}}{(1-\alpha)m_{l,ss}+\alpha(1-m_{l,ss})}(1+(p_l/p_f)^\sigma)((1+(p_l/p_f)^{-\sigma})) \\ &= 1+\frac{2}{\sigma\theta}\frac{1}{1-\alpha}\end{aligned} \tag{B.65}$$

$$\begin{aligned}1+\frac{1}{\varepsilon_{\Delta_j^f}} &= 1+\frac{1}{\sigma\theta}\frac{(1+(p_l/p_f)^\sigma)(1+(p_l/p_f)^{-\sigma})}{\frac{m_{l,ss}}{1-m_{l,ss}}(1-\alpha)+\alpha+\frac{1}{2}(1-\alpha)(1+(p_l/p_f)^\sigma)(1+(p_l/p_f)^{-\sigma})} \\ &= 1+\frac{2}{\sigma\theta}\end{aligned} \tag{B.66}$$

[30] Note that this holds more generally since $\partial\mathcal{M}_l/\partial m_l < 1$.

This conjecture is verified if and only if $\mu_l A_l^{-1} = \mu_f A_f^{-1}$ or

$$\begin{aligned}
\frac{A_l}{A_f} &= \frac{1 + (1-\tilde{\beta})\frac{2}{\sigma\theta}\frac{1}{1-\alpha}}{1 + (1-\beta)\frac{2}{\sigma\theta}} \\
&= 1 + \frac{(1-\tilde{\beta})\frac{\alpha}{1-\alpha} + (\beta - \tilde{\beta})}{\sigma\theta/2 + (1-\beta)} \\
&= 1 + \frac{(1-\beta)\alpha + \frac{1}{2}\beta\theta}{\sigma\theta/2 + (1-\beta)}\frac{1}{1-\alpha}
\end{aligned} \tag{B.67}$$

where $\beta - \tilde{\beta} = \frac{1}{2}\beta\theta > 0$. Hence, there exists a unique productivity gap such that exact neutrality holds. This is in contrast to the previous model in which the steady state customer base adjusts as to equate prices in equilibrium, $p_l = p_f$. This force is still present here but incentives must be such that the m_l that equates prices is exactly $m_{l,ss} = \alpha$.

Note that even if $\alpha \to 0$ the granular seller, for exact neutrality to hold, is required to have a productivity lead. This is since the granular seller discounts by $\tilde{\beta}$ and the fringe seller by β.

□

C Additional Results

C.1 Strategic Interactions

Following Amiti et al. (2019) the key object of interest will be the markup functions $\mu_l(p_l, \hat{p}_f)$ and $\mu_f(p_f, \hat{p}_l)$ with some abuse of notation.[31] Then, following Amiti et al. (2019) and noting that only relative prices enter the markup functions we can derive

$$d \log p_l = \frac{1}{1+\Gamma_l} d(\log w - \log A_l) + \frac{\Gamma_l}{1+\Gamma_l} d \log p_f \tag{C.1}$$

$$d \log p_f = \frac{1}{1+\Gamma_f} d(\log w - \log A_f) + \frac{\Gamma_f}{1+\Gamma_f} d \log p_l \tag{C.2}$$

where $\Gamma_l = -\frac{\partial \log \mu_l}{\partial \log p_l}$ and $\Gamma_f = -\frac{\partial \log \mu_f}{\partial \log p_f}$.

Granular Seller

Recall (2.12). Then,

$$\log \mu_l = \log \left(\kappa_1 + \kappa_2 e^{\sigma \log p_l - \sigma \log p_f} + \kappa_3 e^{\sigma \log p_f - \sigma \log p_l} \right) \tag{C.3}$$

where $\kappa_1 = \frac{\sigma+1}{\sigma} + \frac{1}{\sigma}\frac{1-\hat{\theta}}{\hat{\theta}}$, $\kappa_2 = \frac{1}{\sigma\hat{\theta}}\frac{1-\hat{\theta}}{2}$ and $\kappa_3 = \frac{1}{\sigma\hat{\theta}}\frac{1+\hat{\theta}}{2}$. Then,

$$\Gamma_l = \frac{\sigma}{\mu_l} \left(\kappa_3 (p_f/p_l)^\sigma - \kappa_2 (p_l/p_f)^\sigma \right) \tag{C.4}$$

$$1 + \Gamma_l = \mu_l^{-1} \left[\kappa_1 + (1-\sigma)\kappa_2 (p_l/p_f)^\sigma + (1+\sigma)\kappa_3 (p_f/p_l)^\sigma \right] > 0 \tag{C.5}$$

where the last inequality follows from $\sigma < 1$. It follows that $\Gamma_l > 0$ whenever $p_f > (\kappa_2/\kappa_3)^{1/2\sigma} p_l = \left(\frac{1-\hat{\theta}}{1+\hat{\theta}}\right)^{1/2\sigma} p_l$. From (C.1) it follows immediately that for the granular seller prices are strategic *complements* whenever $p_f < \left(\frac{1-\hat{\theta}}{1+\hat{\theta}}\right)^{1/2\sigma} p_l$ and strategic *substitutes* otherwise (sufficient but not necessary).

Fringe Sellers

Recall (2.13). Then,

$$\log \mu_f = \log \left(\bar{\kappa}_1 + \frac{1}{\sigma\theta} \frac{\varphi + \phi e^{\sigma \log p_l - \sigma \log p_f} + (1-\phi) e^{\sigma \log p_f - \sigma \log p_l}}{1 + (\varphi/2) e^{\sigma \log p_l - \sigma \log p_f} + (\varphi/2) e^{\sigma \log p_f - \sigma \log p_l}} \right) \tag{C.6}$$

where $\bar{\kappa}_1 = \frac{\sigma+1}{\sigma} + \frac{1}{\sigma}\frac{1-\theta}{\theta}$, $\varphi = \frac{1-m_l}{1+m_l}$ and $\phi = \frac{1}{2} + \frac{m_l \theta}{1+m_l}$.

[31] Technically, we would have $\mu_j(p_j, \hat{p}_f, \hat{p}_l)$ but we aggregate to the symmetric equilibrium already.

It follows that

$$
\begin{aligned}
\Gamma_f \propto\ & (\phi D-(\varphi/2)N)(p_l/p_f)^{\sigma}+((\varphi/2)N-(1-\phi)D)(p_l/p_f)^{-\sigma} \\
=& -\left[\frac{\varphi^2}{2}-\phi+\frac{\varphi}{2}(1-2\phi)(p_l/p_f)^{-\sigma}\right](p_l/p_f)^{\sigma}+\left[\frac{\varphi^2}{2}-(1-\phi)+\frac{\varphi}{2}(2\phi-1)(p_l/p_f)^{\sigma}\right](p_l/p_f)^{-\sigma} \\
=& \frac{m_l}{(1+m_l)^2}\left[2\theta(1-m_l)+(2+\theta(1+m_l))(p_l/p_f)^{\sigma}-(2-\theta(1+m_l))(p_l/p_f)^{-\sigma}\right] \qquad \text{(C.7)}
\end{aligned}
$$

where $N=\varphi+\phi(p_l/p_f)^{\sigma}+(1-\phi)(p_l/p_f)^{-\sigma}$ and $D=1+(\varphi/2)((p_l/p_f)^{\sigma}+(p_l/p_f)^{-\sigma})$. Prices are then strategic complements if $p_l>\left(\frac{2\sqrt{1-\theta^2 m_l}-\theta(1-m_l)}{2+\theta(1+m_l)}\right)^{1/\sigma}p_f$.

It follows that prices both for the granular and for fringe sellers prices are strategic complements if $\left(\frac{2\sqrt{1-\theta^2 m_l}-\theta(1-m_l)}{2+\theta(1+m_l)}\right)^{1/\sigma}<\frac{p_l}{p_f}<\left(\frac{1+\hat{\theta}}{1-\hat{\theta}}\right)^{1/2\sigma}$.The interval converges to $\left(\left(\frac{2\sqrt{1-m_l}-(1-m_l)}{3+m_l}\right)^{1/\sigma},\left(\frac{2-m_l}{m_l}\right)^{1/2\sigma}\right)$ as $\theta\to 1$ and to $\left(\frac{1}{3^{1/\sigma}},\infty\right)$ as $m_l\to 0$ jointly with $\theta\to 1$.

Equilibrium

Suppose that relative prices lie within the interval, then in equilibrium comparative statics are given by

$$
d\log\frac{p_l}{w/A_l}=\frac{\Gamma_l}{1+\Gamma_l+\Gamma_f}d\log(A_l/A_f) \qquad \text{(C.8)}
$$

$$
d\log\frac{p_f}{w/A_f}=\frac{\Gamma_f}{1+\Gamma_l+\Gamma_f}d\log(A_f/A_l) \qquad \text{(C.9)}
$$

and markups are increasing in productivity. Further, only changes to relative productivity matter and not each productivity separately.

Change in Granularity

Let $\Gamma_{m,l}=\frac{\partial\log\mu_l}{\partial m_l}$ and $\Gamma_{m,f}=\frac{\partial\log\mu_f}{\partial m_l}$. Then, the extended comparative statics read

$$
d\log\frac{p_l}{w/A_l}=\frac{\Gamma_l}{1+\Gamma_l+\Gamma_f}d\log(A_l/A_f)+\frac{(1+\Gamma_f)\Gamma_{m,l}+\Gamma_l\Gamma_{m,f}}{1+\Gamma_l+\Gamma_f}dm_l \qquad \text{(C.10)}
$$

$$
d\log\frac{p_f}{w/A_f}=\frac{\Gamma_f}{1+\Gamma_l+\Gamma_f}d\log(A_f/A_l)+\frac{(1+\Gamma_l)\Gamma_{m,f}+\Gamma_f\Gamma_{m,l}}{1+\Gamma_l+\Gamma_f}dm_l \qquad \text{(C.11)}
$$

where

$$
\Gamma_{m,l}=\frac{1}{\mu_l}\frac{1}{\sigma\theta(1-m_l)^2}\left[1+\frac{1}{2}((p_l/p_f)^{\sigma}+(p_l/p_f)^{-\sigma})\right] \qquad \text{(C.12)}
$$

$$
\Gamma_{m,f}=\frac{1}{\mu_f}\frac{2}{\sigma\theta}\frac{\frac{1}{2}((p_l/p_f)^{\sigma}+(p_l/p_f)^{-\sigma})-1+\theta\frac{1}{2}((p_l/p_f)^{\sigma}-(p_l/p_f)^{-\sigma})}{[(1+m_l)+(1-m_l)\frac{1}{2}((p_l/p_f)^{\sigma}+(p_l/p_f)^{-\sigma})]^2}\left[1+\frac{1}{2}((p_l/p_f)^{\sigma}+(p_l/p_f)^{-\sigma})\right] \qquad \text{(C.13)}
$$

We note that $\Gamma_{m,l} > 0$ unambiguously. On the other, hand $\Gamma_{m,f} > 0$ if $p_l > p_f$. Hence, if $1 \leqslant \frac{p_l}{p_f} \leqslant \left(\frac{1+\hat{\theta}}{1-\hat{\theta}}\right)^{1/2\sigma}$ then all partial derivatives of the log markup function are positive and equilibrium markups are increasing in the granularity of the large seller. We further note that the fringe seller is relatively less responsive to changes to the granularity of the large seller. This can be verified numerically but intuition can be gained form the limiting case in which $p_l = p_f$.

C.2 Stackelberg Competition

Static Model

Suppose that the large seller acts as a Stackelberg leader and takes the reaction function of fringe sellers into account. Then, the FOC reads

$$\frac{p_l}{w/A_l} = 1 + \frac{1}{-\frac{\partial \log \mathcal{M}_l}{\partial \log p_l} - \frac{\partial \log \mathcal{M}_l}{\partial \log p_f}\frac{d \log p_f}{d \log p_l}} \tag{C.14}$$

As $\mathcal{M}_l(p_l, p_f) = \mathcal{M}_l(p_l/p_f)$ it follows that $\frac{\partial \log \mathcal{M}_l}{\partial \log p_l} = -\frac{\partial \log \mathcal{M}_l}{\partial \log p_f}$. Further, in Appendix C.1 we defined $\frac{d \log p_f}{d \log p_l} = \frac{\Gamma_f}{1+\Gamma_f}$ and discussed when the derivative is positive. Then,

$$\begin{aligned}\frac{p_l}{w/A_l} &= 1 + \left[-\frac{\partial \log \mathcal{M}_l}{\partial \log p_l}\right]^{-1}(1+\Gamma_f) \\ &= \frac{\sigma+1}{\sigma} + \frac{1-\hat{\theta}+\Gamma_f}{\sigma\hat{\theta}} + \frac{1+\Gamma_f}{\sigma\hat{\theta}}\left(\frac{1-\hat{\theta}}{2}(p_l/\hat{p}_f)^{\sigma} + \frac{1+\hat{\theta}}{2}(p_l/\hat{p}_f)^{-\sigma}\right)\end{aligned} \tag{C.15}$$

Hence, for all $\frac{p_l}{p_f} > \left(\frac{2\sqrt{1-\theta^2 m_l} - \theta(1-m_l)}{2+\theta(1+m_l)}\right)$ we know from Appendix C.1 that $\Gamma_f > 0$ and hence the granular seller charges a higher markup under Stackelberg competition.

We can also rewrite the optimal markup as

$$\frac{p_l}{w/A_l} = \frac{1+\sigma\theta(1-\tilde{m}_l)}{\sigma\theta(1-\tilde{m}_l)} + \frac{1+\Gamma_f}{\sigma\hat{\theta}}\left(\frac{1-\hat{\theta}}{2}(p_l/\hat{p}_f)^{\sigma} + \frac{1+\hat{\theta}}{2}(p_l/\hat{p}_f)^{-\sigma}\right) \tag{C.16}$$

where $\tilde{m}_l = m_l + \frac{\Gamma_f}{1+\Gamma_f}m_l$. The first term again denotes the markup analogous to a one-sector Atkeson & Burstein (2008) model with Stackelberg competition. As in an Atkeson & Burstein (2008) the Stackelberg leader acts as if it had a higher market share (if $\Gamma_f > 0$). The seond term is amplified also. The intuition is straightforward. If the granular seller anticipates the fringe sellers to raise prices in response to a marginal increase in p_l the large seller internalises that they can raise prices while losing a limited number of customers. The reverse holds if prices are strategic substitutes, i.e. $\Gamma_f < 0$.

Dynamic Model

Similar steps to B.8 show that relative to the model in the main text the dynamics are not changed. If the large seller takes the measure λ as exogenous then Stackelberg competition only changes within period incentives discussed above. The problem for the fringe seller is unchanged.

An Analytical Example. Let us consider the steady state if fringe sellers are of identical productivity A_f and the granular seller has productivity A_l. As before, $p_l(m_{l,ss}) = p_f(m_{l,ss})$. Hence,

$$1 + \left[-\frac{\partial \log \mathcal{M}_{l,ss}}{\partial \log p_l} \right]^{-1} (1 + \Gamma_{f,ss}) = 1 + \frac{2(1 + \Gamma_{f,ss})}{\sigma\theta(1 - m_{l,ss})} \tag{C.17}$$

where $\Gamma_{f,ss}$ denotes $\frac{d \log p_f}{d \log p_l}|_{p_l = p_l(m_{l,ss}), p_f = p_f(m_{l,ss}), m_l = m_{l,ss}}$ evaluated at the steady state.

For the identical reasons as in B.6 it follows that

$$\mu_l = 1 + \frac{2(1-\beta)(1 + \Gamma_{f,ss})}{\sigma\theta(1 - m_{l,ss})} \tag{C.18}$$

$$\mu_f = 1 + \frac{2(1-\beta)}{\sigma\theta} \tag{C.19}$$

Equating $\mu_l \frac{w}{A_l} = \mu_f \frac{w}{A_f}$, we can show that, if a steady state exists, then $m_{l,ss}$ solves

$$\frac{m_{l,ss} + \Gamma_{f,ss}}{1 - m_{l,ss}} = (A_l / A_f - 1) \left[1 + \frac{\sigma\theta}{2(1-\beta)} \right] \tag{C.20}$$

and with markup

$$\mu_l = \left(1 + \frac{2(1-\beta)}{\sigma\theta} \right) \frac{A_l}{A_f} \tag{C.21}$$

and hence Stackelberg competition only changes the steady state customer base, leaving steady state prices and markups unchanged. This again highlights the discipline imposed by the market on the granular seller. Hence, a granular seller that acts as a Stackelberg leader cannot break the neutrality result.

C.3 Law of Motion of Distribution λ

Figure C.1 plots the evolution of next period's density λ' as a function of p_l given that fringe sellers charge their equilibrium price p_f and given that the initial customer base of the large seller is at its steady state $m_{l,ss}$. Results have been found numerically to generalise to arbitrary m_l and p_f. Changes in the price of the large seller only has negligible effects on the distribution λ', providing suggestive validation for the partially oblivious solution concept used in the paper. In a neighbourhood around its steady state price the granular seller cannot move the distribution of any productivity level by more than 2%. Further, the density for the most extreme productivity types are most sensitive. This implies that the absolute deviations due to pricing are even smaller, e.g. in the order of 0.01 to 0.075 percentage points.

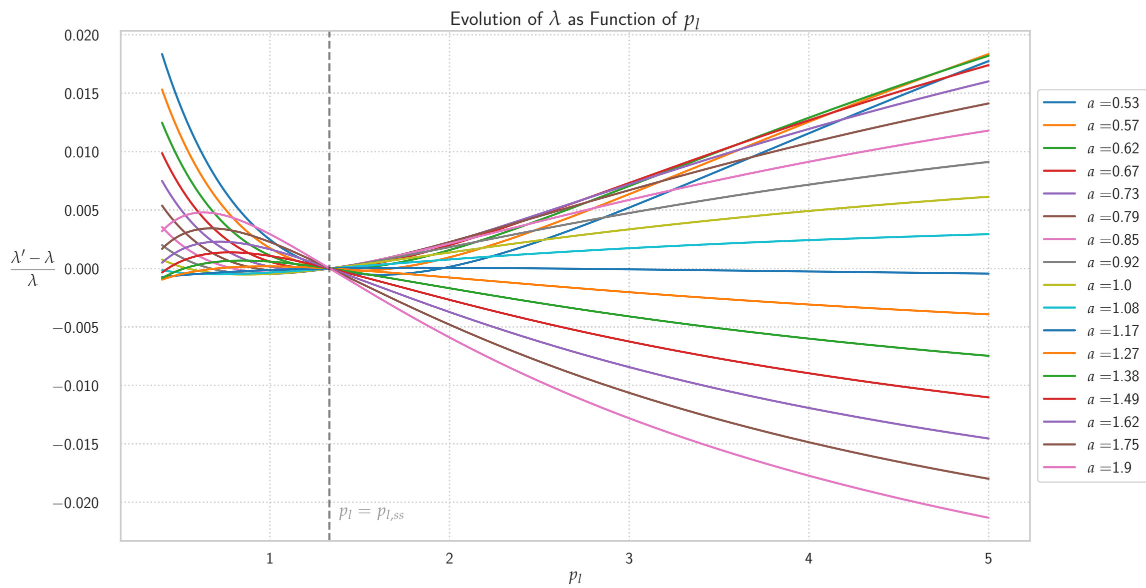

Figure C.1: Stationary Equilibrium.

Intuition. Consider two prices of the large seller p_1 and p_2. Denote by $(\lambda_1'(a), m_1)$ and $(\lambda_2'(a), m_2)$ next period's distribution and large seller's customer base under price p_1 and p_2, respectively. Let $\Delta_k(\tilde{a}) =$

$\Delta_j^f(p_f(\tilde{a}), p_f, p_k)$ for $k = 1, 2$. Then,

$$\begin{aligned}
\lambda_1'(a) - \lambda_2'(a) &= (1 - m_l) \int P(a|\tilde{a})\lambda(\tilde{a}) \left(\frac{\Delta_1(\tilde{a})}{1 - m_1} - \frac{\Delta_2(\tilde{a})}{1 - m_2} \right) d\tilde{a} \\
&= (1 - m_l)\pi(a) \underbrace{\int \lambda(\tilde{a}) \left(\frac{\Delta_1(\tilde{a})}{1 - m_1} - \frac{\Delta_2(\tilde{a})}{1 - m_2} \right) d\tilde{a}}_{=0} \\
&\quad + (1 - m_l) \int (P(a|\tilde{a}) - \pi(a))\lambda(\tilde{a}) \left(\frac{\Delta_1(\tilde{a})}{1 - m_1} - \frac{\Delta_2(\tilde{a})}{1 - m_2} \right) d\tilde{a} \\
&= \mathbb{C}ov_\pi \left(P(a|\tilde{a}), \left(\frac{\Delta_1(\tilde{a})}{1 - m_1} - \frac{\Delta_2(\tilde{a})}{1 - m_2} \right) \mathbb{E}[m|\tilde{a}] \right) \qquad \text{(C.22)}
\end{aligned}$$

where $\mathbb{C}ov_\pi(f(a), g(a)) \equiv \int f(a)g(a)\pi(da)$ and the last line uses $(1 - m_l)\frac{\lambda(a)}{\pi(a)} = \int m\Lambda(a, m)dm =: \mathbb{E}[m|a]$. It is immediate that if the productivity process was iid, i.e. $P(a|\tilde{a}) = \pi(a)$, then the approximation is exact, i.e. $\lambda_1' - \lambda_2' = 0$. On the other extreme suppose that $P(a|\tilde{a}) = \mathbf{1}(\tilde{a} = a)$, i.e. persistence of one, then $\lambda_1'(a) - \lambda_2'(a) = \left(\frac{\Delta_1(a)}{1 - m_1} - \frac{\Delta_2(a)}{1 - m_2} \right) \lambda(a)$. Even in the perfect persistence case the approximation seems good. To see why, suppose that $p_1 > p_2$ is relatively high such that $\Delta_1(a) > \Delta_2(a)$ for all a. But at the same time $1 - m_1 > 1 - m_2$ if $p_1 > p_2$. This offsets some of the change.

This provides us with the main insight. The pricing decision of a granular seller feeds through to the evolution of the measure λ via the difference $\frac{\Delta_1(\tilde{a})}{1 - m_1} - \frac{\Delta_2(\tilde{a})}{1 - m_2}$. Since $\int \Delta_k(a)\lambda(da) = 1 - m_k$ for $k = 1, 2$ by definition, this effect is zero on average. So persistence in the productivity process is necessary such that the effect does not average out.

D Additional Figures

D.1 Transition Dynamics

Figure D.1 expands on the discussion of section 4.2. The bottom panels highlight three points. First, the bottom left panel shows that relative prices $p_l/p_f(a)$ are small when the granular seller is small and rise along the transition. However, the bottom middle panel shows that the mapping from relative prices to net flows between fringe sellers and granular seller is more intricate. Even though relative prices are the lowest early on net flows in absolute terms are increasing over time. This is due to the fact that net flows between fringe seller of productivity a and the granular seller are given by the product of the customer base of the large seller and a function of relative prices, i.e. $\theta m_l \frac{1-(p_f(a)/p_l)^\sigma}{1+(p_f(a)/p_l)^\sigma}$. As an implication, the pricing of the granular seller does not impact pricing decisions of fringe sellers until the granular seller has reached sufficient granularity.

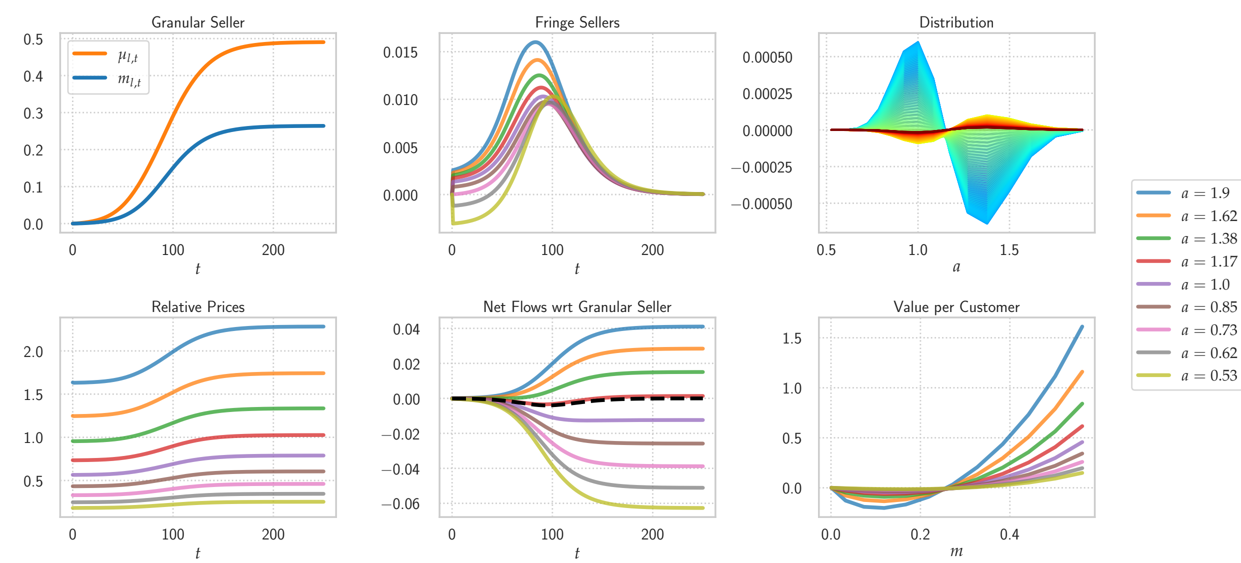

Figure D.1: Transition Dynamics after Introduction of Granular Seller.

The bottom right panel plots $V_j(a,m) - V_j(a,0)$ as a function of $m \in [0, 0.6]$ for values of productivity a. The non-monotonicity of the value function in m is key for understanding the dynamics. For low values of m the granular seller charges a price below the stationary equilibrium price. A marginal increase in m_l amplifies the lower relative price in terms of net flows. For unproductive firms this implies greater outflows to the large seller. For productive firms this implies smaller inflows relative to the stationary equilibrium as well as relative to other fringe sellers. Hence, it is harder for fringe firms to attract new customers. This reduces the intertemporal investment motive and fringe sellers raise their price. The effect is reversed when $m > m_{l,ss}$ as it becomes easier to attract customers. The forces balance exactly when $m = m_{l,ss}$.

D.2 Firm Growth with Alternative Matching Process.

Suppose the matching process in (4.3). Figure (D.2) plots the growth rate of fringe sellers relative to the counterfactual of no large seller, $g - g_{nls}$, as a function of productivity a. The larger α the greater the deviations from the benchmark of no large seller. For example, for $\alpha = 0.5$ the most productive firms grow by more than 3 percentage points less than if there was no granular seller while the least productive firms shrink by 4 percentage points less. This has natural implications for the reallocation of customers across fringe sellers and measures of efficiency.

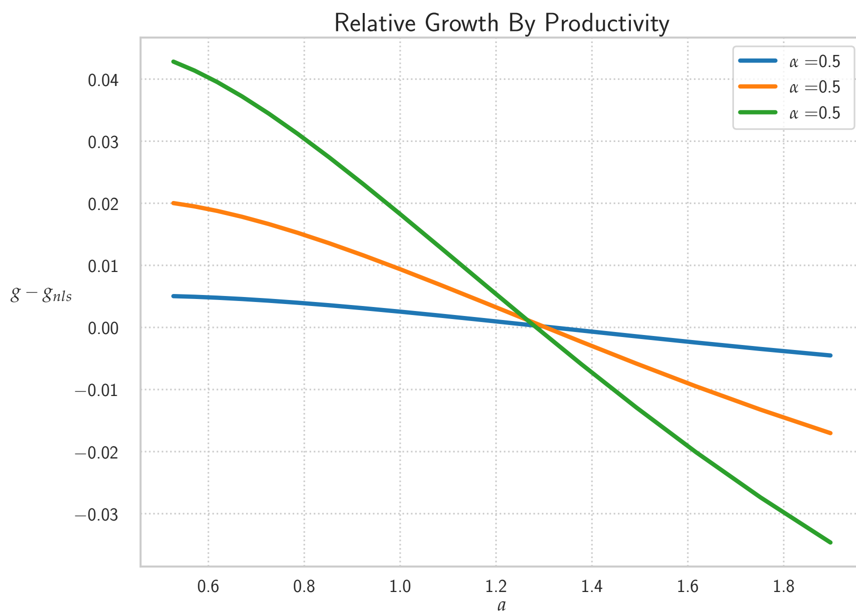

Figure D.2: Non-Neutrality of Firm Growth in the Presence of a Granular Seller

E Decomposition

Denote by V_l,V_j,p_l, p_f, and Δ_j^f the equilibrium value and policy functions as well as the law of motion for the customer base of a fringe seller when matching follows (2.2) and $V_l^\dagger$,$V_j^\dagger$,$p_l^\dagger$, $p_f^\dagger$, and $\Delta_j^{f,\dagger}$ the equilibrium value and policy functions as well as the law of motion for the customer base of a fringe seller when matching follows (4.3). Then, we can compute the following counterfactuals.

(a) The benchmark model in which matching follows (2.2)

$$\tilde{p}_f^{(1)} = \arg\max_p \Delta_j^f(p, p_f, p_l) \left[(p - \frac{w}{a}) \frac{I}{p} + \beta \int V_j(a', m_{l,ss}) P(da'|a) \right] \tag{E.1}$$

(b) The alternative model in which matching follows (4.3)

$$\tilde{p}_f^{(2)} = \arg\max_p \Delta_j^{f,\dagger}(p, p_f^\dagger, p_l^\dagger) \left[(p - \frac{w}{a}) \frac{I}{p} + \beta \int V_j^\dagger(a', m_{l,ss}^\dagger) P(da'|a) \right] \tag{E.2}$$

(c) The hypothetical counterfactual in which we only adapt the matching process from (2.2) to (4.3), holding fixed the competitor's prices, the size distribution and continuation value. That is

$$\tilde{p}_f^{(3)} = \arg\max_p \Delta_j^{f,\dagger}(p, p_f, p_l) \left[(p - \frac{w}{a}) \frac{I}{p} + \beta \int V_j(a', m_{l,ss}) P(da'|a) \right] \tag{E.3}$$

This counterfactual isolates the fact that under the alternative matching process (4.3) fringe firms are harder to find – all else held constant.

(d) The hypothetical counterfactual in which we maintain the matching process (2.2) but adapt the equilibrium prices of competitors. That is

$$\tilde{p}_f^{(4)} = \arg\max_p \Delta_j^f(p, p_f^\dagger, p_l^\dagger) \left[(p - \frac{w}{a}) \frac{I}{p} + \beta \int V_j(a', m_{l,ss}) P(da'|a) \right] \tag{E.4}$$

This counterfactual isolates the strategic interaction between pricing decisions. In particular, it computes the effect on the pricing of fringe sellers when overall prices are higher in the economy.

The counterfactuals are all plotted in the left panel of Figure E.1. The right panel plots the value functions $V_j(a', m_{l,ss})$ and $V_j^\dagger(a', m_{l,ss}^\dagger)$. The distance $\tilde{p}_f^{(2)} - \tilde{p}_f^{(1)}$ restates the deviation from approximate neutrality. Note that $\tilde{p}_f^{(3)} - \tilde{p}_f^{(1)} > 0$ for all a. This captures the intuition that as fringe sellers are harder to find they are less able to attract new customers and raise prices to exploit their own customer base. At last, the difference $\tilde{p}_f^{(4)} - \tilde{p}_f^{(1)} > 0$ captures the fact that prices are strategic complements for high and strategic substitutes for low levels of productivity a. The right panel shows that as the granular seller charges significantly higher prices under (4.3) than under (2.2) the per customer value of a fringe seller is higher under the former matching process. This introduces partially offsetting incentives to lower prices.

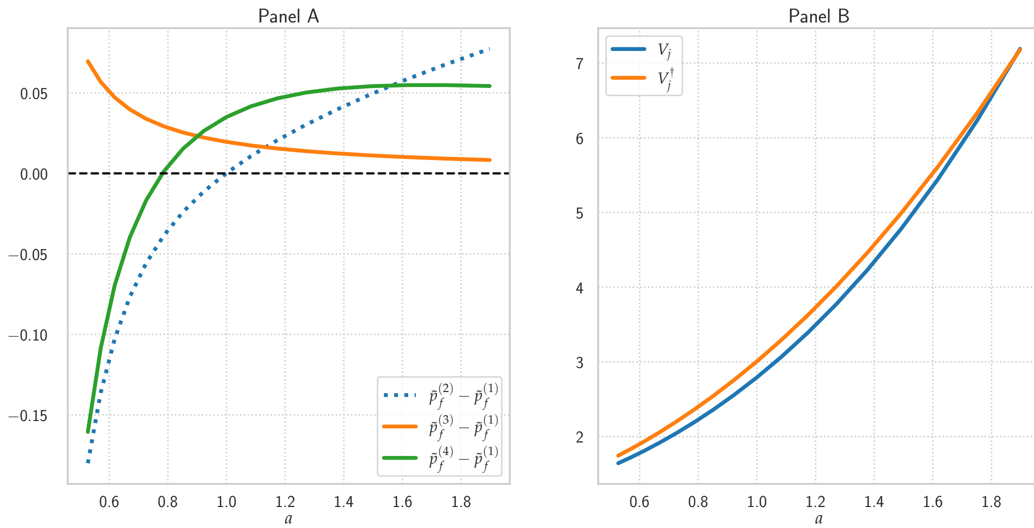

Figure E.1: Decomposition

F Computational Appendix

F.1 Algorithm for Stationary Equilibrium

I use value function iteration to compute the stationary equilibrium of the economy.

(1) Construct a discretised grid for productivity and the customer base of the large seller. I construct the productivity grid using the Rouwenhorst (1995) method with $N_a = 17$ grid points. Denote the Markov transition matrix by P. I use $N_m = 15$ grid points for the customer base of the large seller to construct a non-uniform grid. The grid is more concentrated at lower values for m_l.

(2) Guess the value functions of the large and fringe sellers, V_l and V_j, on the grid, the Markov perfect pricing strategies p_l and p_f, the steady state customer base of the large seller $m_{l,ss}$ and the distribution λ as well as aggregate profits Π. We normalise the wage to one.

(3) Solve the optimal pricing decision of the granular seller given a guess for $V_l^{(n)}$, $p_f^{(n)}$, $m_{l,ss}^{(n)}$, and $\lambda^{(n)}$. I use a vectorised Golden Search algorithm to maximise the objective and cubic interpolation to evaluation points off the grid. Denote the maximiser by $\hat{p}_l^{(n)}$. Given $\hat{p}_l^{(n)}$ and the guess $p_f^{(n)}$ iterate on the law of motion of the large seller's customer base until convergence and update the steady state customer base to get $m_{l,ss}^{(n+1)}$.

(4) Solve the optimal pricing decisions of fringe sellers given a guess for $V_j^{(n)}$, $\lambda^{(n)}$ and updated guesses for $m_{l,ss}^{(n+1)}$ and $\hat{p}_l^{(n)}$. Obtain a new maximising $\hat{p}_f^{(n)}$. Given $\hat{p}_l^{(n)}$, $\hat{p}_f^{(n)}$, and $m_{l,ss}^{(n+1)}$ use the Law of motion of the distribution to get an update $\lambda^{(n+1)}$. Using the policy functions we can obtain a new value for aggregate profits $\Pi^{(n+1)}$ and hence $I^{(n+1)}$.

(5) Check the distance between policy functions, value functions and profits from previous guesses. If the distance is not below the tolerance level, set $p_l^{(n+1)} = \omega \hat{p}_l^{(n)} + (1-\omega) p_l^{(n)}$ and $p_f^{(n+1)} = \omega \hat{p}_f^{(n)} + (1-\omega) p_f^{(n)}$ for a dampening parameter $\omega \in (0,1)$. Use these new policy functions to update the value functions $V_l^{(n+1)}$ and $V_j^{(n+1)}$.

F.2 Algorithm for Transition Dynamics

In the partially oblivious equilibrium the customer base of the large seller completely characterises the aggregate state. As the solution to the stationary equilibrium already gives policy functions with m_l as an argument we can compute transition dynamics by interpolating our discretised policy functions. To confirm that the partially oblivious equilibrium is a good approximation we check that the distribution is approximately constant along the transition.